Introduction to the theory for optimal design of satellite constellations for earth periodic coverage

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Abstract

Design of satellite constellations is associated with continuous or periodical coverage of an Earth region of interest depending on the concrete task of the space mission. Satellite constellations for continuous and periodical coverage are principally differed by orbit parameters as well as by methods of constellation design. In this study route theory for design of constellations for circular low Earth orbits providing periodical coverage with minimum revisit time criteria is described. The route theory includes: 1) consideration of so called route orbital pattern as mathematical abstraction of arbitrary constellation; 2) analytical solution for calculation problem of distribution of revisit time values on Earth surface for one satellite and multi-satellite route pattern; 3) formulating several regularities for revisit time as a function of satellites positions in constellation; 4) development of method for optimal design of constellation under given criterion and requirements. Using of route orbital patterns is practically not connected with restriction for type of constellation. It is also shown that optimal constellations calculated using the route theory are in general not worse and in many cases much better (in respect of criteria mentioned above) comparing with the analogs found using other well-known methods for periodical coverage.

1. History of the problem

From the very beginning of the space age the Earth artificial satellites serve the mankind. Moreover, the satellite importance for the human being was emphasized when group of satellites functioning together to solve common task, had been created. These groups are commonly called satellite constellations. Efficiency of such groups functioning depends on how correct these satellites orbits are chosen, and how the satellites relative orbital movement is organized. This problem is discussed below.

The way to solve the problem to optimize the satellite constellations was outlined in principle in the 60th – 70th, when the importance of study of so called function of satellite coverage (continuous or periodic) of the Earth was recognized [1-11]. This function allows interpreting the purposes of big number of satellite systems achieved by means of different on-board equipment, in unified abstract aspect. Namely, this aspect took the form of establishing (continuous or discontinuous, respectively) necessary geometric conditions of relative positioning of satellites and Earth surface objects observed.

Important theoretical generalization of ballistic design methods for satellite constellations for continuous and periodic coverage known by the late 70th, is presented in [11]. The most valuable result was obtained, independently and almost at once, by Walker [5, 6, 9, 10] and Mozhaev [7, 8]. This result yielded in the development and comprehensive grounding of so called symmetrical constellations for continuous coverage problem. Each symmetrical constellation corresponds to a certain code of symmetry describing relative satellite disposition in the network. In Russian literature the symmetrical constellations are called "kinematically regular networks", in English – "Walker constellations". Later the symmetrical constellation properties for the continuous coverage problem were subjected to close inspection of both these constellations inventors – Walker and Mozhaev, and other authors (see, for example, [12-15]).

Some other methods to optimize the continuous coverage satellite constellations were introduced later on. These are the "streets of coverage" method [16, 17], original Draim's method to analyze constellations using elliptical orbits with minimum needed number of satellites [18], and others. However, such particular methods involve the essential narrowing the classes of constellation in use, and are not universal and simple like symmetrical constellation method. Even nowadays most of investigators of this problem refer first to symmetrical constellations at designing the continuous coverage satellite system.

Simplicity of continuous coverage system design on the base of symmetrical constellations did not allow to fully recognize the complexity of periodic coverage problem, although it was emphasized by some investigators both many years ago [15], and not long ago [26]. Mozhaev, in particular, points to that the main criterion function in the periodic coverage problem (revisit time) is not differentiable (as in the continuous coverage problem), but "even discontinuous and unable to be set by formulas" [15]. It can be spelled out that mathematics did not offer any ready method able to be directly applied to optimize satellite constellations for periodic coverage. Moreover, the word "optimum" was conditional in the periodic coverage problems down to recent times. Usually, this word referred to rational (from different points of view) alternatives of satellite constellations chosen in a priori predetermined narrow classes of orbital structures.

In order to provide equal precession of satellite orbits the Earth surface coverage satellite constellations are usually organized on circular orbits of equal radius and inclination. In this context it can be pointed to some practically validated narrowing of the optimization range of satellite constellations for periodic coverage (as compared with those for continuous coverage) that is defined by applying for periodic coverage relatively low circular orbits up to about 5,000 km (this maximum altitude can vary depending on available width of the satellite coverage swaths). However, the said circumstance does not simplify, if not complicates, the problem under consideration.

Due to the complexity of the periodic coverage problem the first attempts to solve it were reduced to a simple application of symmetrical constellations method. The latter approved itself at solving the continuous coverage problem, despite that seldom some other particular methods were used [15, 19-26]. As a result, symmetrical constellations have been widely propagated in practice of satellite constellation design not only for continuous but also for periodic Earth coverage, lacking something better.

However, soon afterwards the Soviet (Russian) authors have revealed the versions of satellite constellation for periodic coverage with “poor” symmetry. These constellations badly fit in symmetrical constellation theory but stably have the better or the same periodicity values comparing with symmetrical constellations [27-37]. Unfortunately, judging by the publications, these results were overlooked and not assessed by the world scientific society till nowadays. ”The literature on low-altitude satellite constellations that provide non-continuous coverage of a small part of the globe is sparse”, authors of [23] pointed out. Really, there were many Russian-language papers devoted to the subject of periodic coverage of the Earth surface areas. Obviously, the reason was that far from all Russian-language papers dealing with the synthesis of periodic coverage orbital structures were translated into English.

As a matter of fact, so called secure constellations were revealed and substantiated in the mentioned papers [27-37]. Secure constellation is designed by means of N satellites (or N identical frontal satellite groups) disposition along common trace(s) on the Earth surface with time lag equal to the 1/N of maximum revisit time provided for a certain Earth region by one of N satellites (by one of N frontal satellite groups) of the constellations. Secure constellation provides maximum revisit time which is equal to the said time lag in the movement of satellites (frontal satellite groups) along the trace(s).

Frontal satellite group consists of satellites whose sub-satellite points at any moment have the same latitude and are shifted relative one another by longitude only. It should be noted that the “secure constellations” term was introduced by the author of present paper. In referenced papers of other Russian authors the said constellations either have other appellations or do not have any.

Steadily good features of secure constellation at the periodic coverage allowed some researchers to throw out a suggestion that these constellations are absolutely optimal in solving the periodic coverage problem: Mozhaev [15, p. 259], Saulsky [29, p. 112].

However, the author had queried this statement in [38], and later he obtained the final negative conclusion about optimality of secure constellations [39, 40]. It was proved by revealing some effective satellite constellations that did not belong to any known class. Revealed constellations were testified to surpass secure constellations in minimum revisit time criterion in a series of different cases.

Therefore, the situation was contradictory: comparative features of different satellite constellation classes essentially depend on the choice of common comparison conditions, and do not allow to prefer one to another in a general case. Author supposes that his route theory for orbital design of periodic coverage satellite constellations, serves to resolve this situation. Fundamentals of the said theory are stated in [41] and are subject of author's lecture courses in MSTU named after Bauman.

For clear reasons only some basic principles of the theory and restricted amount of numerical results are demonstrated in present paper. Only a small part of the theory statements is represented in the paper. Statements represented are given with no proof.

2. Route theory: brief description

Ballistic parameters of the constellations on circular orbits of equal altitude and inclination are specified by vector

$$S_N = \{S_j\},\ S_j = (H, i, \Omega_j, u_j),\ j = 1(1)N, \tag{1}$$

where H – orbit altitude; i – orbit inclination; Ω_j – longitude of the orbit ascending node of j-th satellite; u_j – latitude argument of j-th satellite of the constellation in the initial point of time; N – number of satellites in the constellation.

Components H, i of vector (1) define the orbit types, as well as components Ω_j and u_j specify orbital or phase structure of the constellation:

$$O_N = \{O_j\},\ O_j = (\Omega_j, u_j),\ j = 1(1)N. \tag{2}$$

Traditional approaches to solve the problems of periodic coverage are based on the elaboration of optimization methods with simplifying assumptions reduced generally to a priori preset of the orbital structure narrow classes O_N in the form of symmetrical or other types of constellations. The considered route theory is based on the refuse of satellite constellation optimization within the framework of narrow classes of orbital structures, and on the revealing of the regularities for general set of ballistic and system features of periodic coverage satellite constellations. The system features are understood here as revisit time τ, and swath width Π that is formed on the Earth spherical surface by momentary ranges of the satellites coverage. The revisit time is understood as maximum revisit time provided with a priori preset probability level, and the momentary ranges of the satellites coverage can have different form depending on the type of available on-board facilities for observation.

Classical problem of periodic coverage for area R being observed on the Earth surface consists in search for the minimum of function $\tau(O_N)$ and value O_N^*, where this minimum is reached. The problem is formulated as follows:

Problem A. Given: N, R, H, i, Π.

To find: $$O_N^* = \arg\min_{O_N} \{\tau(O_N) = \tau(O_N / N, R, H, i, n)\} \qquad (3)$$

and $$\tau_{\min} = \tau(O_N^*). \qquad (4)$$

Key point of the route theory (giving the latter its name) is so called route constellation used as a mathematical abstraction for approximation of arbitrary satellite constellation. The satellites in route constellation move on geosynchronous orbits along one or several closed traces (or routes) on the Earth surface with repetition period

$$T_{tr} = m \cdot T_{dr} = n \cdot T_{ef}, \qquad (5)$$

where T_{dr} – draconic orbital period of satellites; T_{ef} – efficient astronomical day or simply day (time span between two subsequent passes of a fixed equator point over ascending node of the satellite orbit); m, n – integer coprime numbers characterizing the number of circuits and number of efficient astronomical days within the repetition period of the satellite traces.

Value m/n is a repetition factor of a geosynchronous orbit. Repetition factor m/n and inclination i unambiguously define the orbit altitude H of route constellation satellites.

Orbital structure of the route constellation is properly characterized by longitudes of the traces start on the equator (trace longitudes) L_j and time positions of satellites on their routes (on-trace positions) t_j, that are in one-to-one conformity with traditional parameters Ω_j, u_j of the orbital structure. Values L_j and t_j vary within the following intervals:

$$L_j \in [0, \Delta L), \; t_j \in [0, T_{tr}) \qquad (6)$$

where

$$\Delta L = 2\pi / m \qquad (7)$$

– distance between neighboring points of intersection with equator of the one satellite trace ascending parts within the repetition period T_{tr} of this trace (*internodal distance*).

Internodal distance (7) is smaller by a factor of integer n than the interval between ascending nodes of the satellite on two subsequent circuits (inter-circuit distance) ΔL_{ic} defined as

$$\Delta L_{ic} = n \cdot \Delta L. \qquad (8)$$

Application of the route constellation is connected with substitution of vectors (1) and (2) by vectors (9) and (10), respectively:

$$S_N = S_{N<2 \cdot N+2>} = \{S_j\}, \quad S_j = (m/n, i, L_j, t_j), j = 1(1)N; \qquad (9)$$

$$O_N = O_{N<2 \cdot N>} = \{O_j\}, \quad O_j = (L_j, t_j), j = 1(1)N. \qquad (10)$$

This substitution, i.e., the assumptions defining the route constellation do not restrict in practice the domain of possible orbital dispositions both in altitude H, inclination i of the satellite orbits, and in relative position of the satellites on these orbits. Let's show that.

Really, it is not difficult to note that the infinite discrete multitude $\{H_g\}$ of geosynchronous orbit altitudes corresponding to various repetition factors m/n and fixed inclination i, corresponds to continuous multitude $\{H\} \supset \{H_g\}$ of all possible values H as the domain of rational numbers (fractions) and real numbers on any closed interval of axis of altitude H. At the same time, it is known that for any real number within the closed interval a rational number can be found being arbitrarily close to it. So, for any fixed values $H = H_0, i = i_0$, a repetition factor m/n of geosynchronous orbit with inclination $i = i_0$ and altitude H being arbitrarily close to H_0, can be found. Hence, the assumptions defining the route constellations do not really restrict the domain of possible satellite orbits. Taking into account that it is permitted in the route constellations that satellites move non-synchronously along arbitrarily located traces, one can conclude that the phase structure restriction in the route system is also absent.

Taking (10) into consideration, the classical problem of periodic coverage is formulated in terms of the route theory as follows:

Problem B. *Given:* N, R, m, n, i, Π .

To find: $$O_N^* = \arg\min_{O_N} \left\{ \begin{array}{l} \tau(O_N) = \\ = \tau(O_N / N, R, m, n, i, n) \end{array} \right\} \qquad (11)$$

and $$\tau_{min} = \tau(O_N^*) . \qquad (12)$$

The proposed method to solve Problem B is based on the fact that in any problem of multidimensional optimization a duality of the target function $\tau(O_N)$ and its definition range $\overline{O}_N$

takes place. In other words, the problem with complex target function can be reformulated in such a way that target function will be easier but the “complexities” will be transferred to the optimization area. A reverse transformation is possible, too. The area $\overline{O}_N$ dimensions define the a priori information about location of optimum O_N^*: the larger is $\overline{O}_N$ the more is a priori information of O_N^* location, but the less difficult is perhaps the search of this point due to simplifying of function $\tau(O_N)$ type.

In our case, “simplifying” of function $\tau(O_N)$ is reached owing to the description of phase structure O_N by vector (10) instead of traditional vector (2). That allows to essentially reduce a priori uncertainty of function $\tau(O_N)$ on parameters L_j. In fact, it is seen from (6) and (7) that range of L_j definition is not large and is 22.5…45 degrees for one-day (n = 1) low orbits about 200…5,000 km, respectively (in this case m varies from 16 to 8). With increasing of the trace repetition period (increasing of n) the range of L_j definition decreases rapidly down to some degrees and degree fractions.

Disadvantage is that a priori uncertainty of parameters t_j rises sufficiently with increasing of n. In fact, as it can be seen from (6), for one-day orbits the range of definition of parameters t_j is about 24 hours, and with growing repetition period of the trace this range increases n-fold while expanding to enormous dimensions at large values of n.

At the same time, a large uncertainty of parameters t_j is with usury compensated by that at the description of orbital structure with expression (10) the route theory reveals a series of regularities of revisit time variation $\tau(O_N)$ as a function of parameters t_j. In its turn, it allows to create specific theoretical and program-algorithmic means for optimizing function $\tau(O_N)$ on parameters t_j.

Due to limited volume of the paper the theoretical statements concerning optimization of optional constellations are not represented. Present paper shows the route theory possibilities on the example of new efficient class of satellite constellations revealed and substantiated within the framework of the route theory. The author calls these satellite constellations as regular constellations.

Regular constellation is N-satellite constellation obtained from one satellite as a basic block by means of its successive (N-1)-fold repetition simultaneously with longitude shift by $\Delta L_{tr} \cdot k, k = 1(1)N-1$, and time shift $\Delta t \cdot k\ ,\ k = 1(1)N-1$ along the satellite trace (Fig. 1).

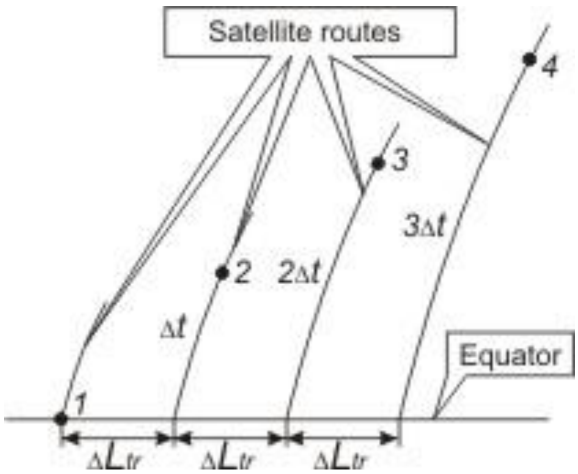

Fig. 1: Relative disposition of the satellites in regular constellation (*N=4)*.

ΔL_{tr} and Δt used in this definition are called, respectively, longitudinal and time shifts of the regular constellation. Reasoning from physical meaning of these values, their variation ranges are determined in the following way:

$$\Delta L_{tr} \in [0, \Delta L),\ \Delta t \in [0, T_{tr}), \qquad (13)$$

and an orbital structure of regular constellations is defined according to formulas:

$$L_j = E[\Delta L_{tr} \cdot (j-1)],$$
$$t_j = E[\Delta t \cdot (j-1)], \qquad (14)$$
$$j = 1(1)N,$$

where function *E* recalculates the argument values for ranges (6) of admissible variation of L_j and t_j magnitudes.

The satellites of regular constellation are shifted along their traces by constant relative value Δt, therefore the determination of satellites positions t_j, j=2(1)N $(t_1 = 0)$ on the traces is reduced to the determination of Δt value providing function $\tau(\Delta t)$ minimum. Taking into account that the definition range of time parameter Δt is large (n efficient astronomical days), this problem solution seems to be highly difficult.

Function $\tau(\Delta t)$ property allowing cardinal simplifying of this problem, consists in the following. For specific area R and a fixed longitudinal shift ΔL_{tr} of N-satellite traces in regular constellation revisit time function $\tau(\Delta t)$ versus time shift Δt meets Lipshitz condition within interval $[0, T_{tr}]$ with constant (N – 1).

It is known that function $f(x)$ satisfies Lipshitz condition in the range $[a,b]$, if there exists such constant Q>0, that $|f(u)-f(v)| \leq Q \cdot |u-v|$ for any $u,v \in [a,b].$ Constant Q is called Lipshitz constant. In our case $f(x) = \tau(\Delta t)$ and Q=N-1. This statement allows to minimize the $\tau(\Delta t)$ function using known mathematical methods for Lipshitz fuctions.

3. Numerical results: comparing of secure and regular constellations

So as to estimate the possibilities of regular constellations it seems to be expedient to compare them with secure systems considered up to the recent time as most preferable for periodic coverage. Equal comparison conditions can be provided by analyzing one-trace alternatives of regular and secure constellations ($\Delta L_{tr} = 0$).

It is easy to see that in the one-trace disposition case secure constellations are a small subset of wide class of regular constellations at $\Delta t = \tau_{max} / N,$ where τ_{max} – one satellite revisit time for the Earth surface specific region. As it was mentioned before secure constellation provides revisit time $\tau_N = \Delta t$ (hereinafter for convenience number N of satellites in the constellation is added as an index to the notation of revisit time τ_N for N-satellite system).

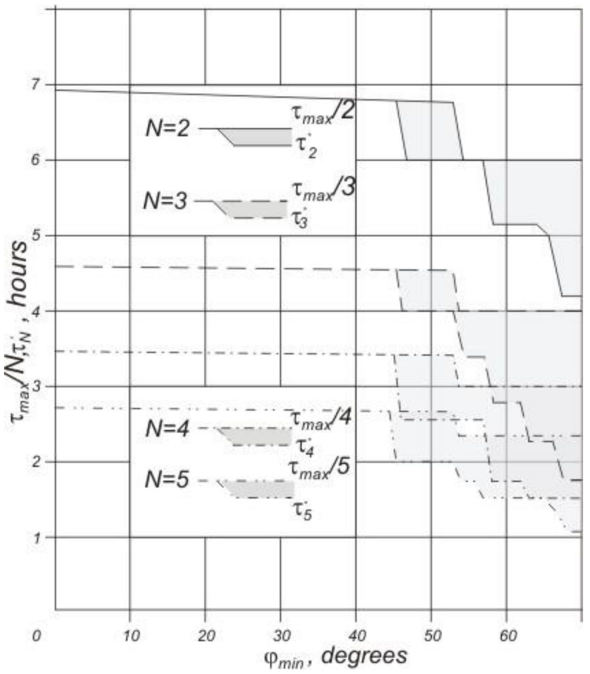

Fig. 2: Periodicities of coverage of Earth latitude belt $\varphi_{\min} ... 70^o$ for secure ($\tau_{\max} / N$) and optimum regular (τ_N^*) constellations versus $\varphi_{\min}$ for the satellite swath *П*=2790 km.

According to [27-37] it can be concluded that the revisit time τ_N^* for optimum regular constellations is always reached on the secure constellations set, and it is $\tau_N^* = \Delta t = \tau_{\max} / N$. Numerical analysis presented below yields that it is really true but in some cases only. Furthermore, application of optimum alternatives of regular constellations in comparison with secure constellations can provide exceptionally large benefit, i.e. value $\tau_{\max} / N - \tau_N^*$.

Example to consider are constellations of $N = \{2,3,4,5\}$ satellites. The said constellations are formed on geosynchronous orbits with repetition factor m/n= 29/2 and inclination $i = 96^o$ (altitude H= 731 km).

Figs. 2-4 show periodicity (revisit time) magnitudes for coverage of Earth latitude belt $(\varphi_{\min}, 70^o)$ for secure ($\tau_{\max} / N$) and optimum regular (τ_N^*) constellations under said restrictions, versus lower limit $\varphi_{\min}$ of latitude belt for the following values of the satellite swath П=2790, 2500, 2000 km.

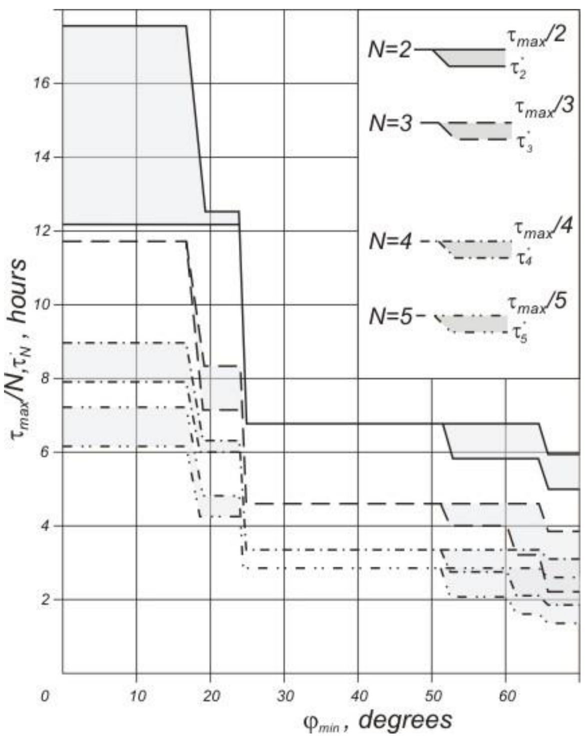

Fig. 3: Periodicities of coverage of Earth latitude belt $\varphi_{\min} ... 70^o$ for secure ($\tau_{\max}/N$) and optimum regular (τ_N^*) constellations versus $\varphi_{\min}$ for the satellite swath *П*=2500 km.

Value of revisit time benefit $\tau_{\max}/N - \tau_N^*$ using optimum regular constellations for various values of $\varphi_{\min}$ in the said figures is defined by the value of shaded range of the line being normal to the abscissa axis at the $\varphi_{\min}$ point.

Figs. 2-4 show that due to using of optimum regular constellations under the above restrictions the revisit time benefit value can reach 5 hours for two-satellite constellations, and 1 hour – for three-, four-, and five-satellite constellations comparing with secure constellations.

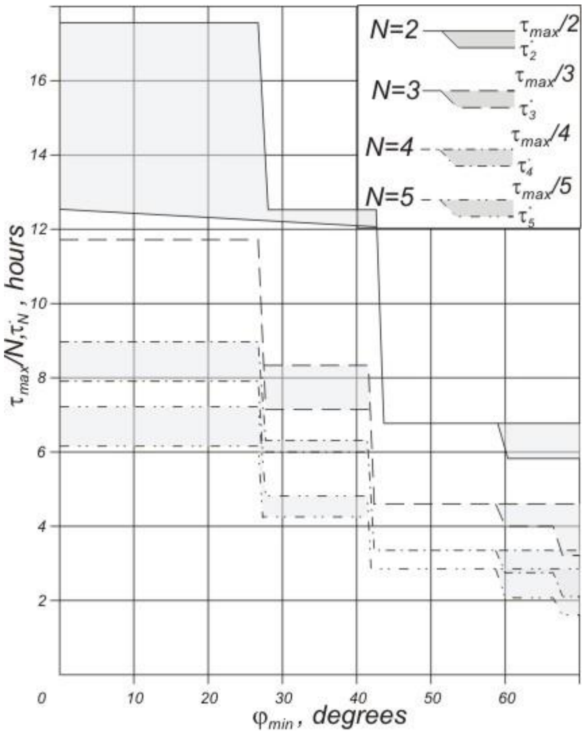

Fig. 4: Periodicities of coverage of Earth latitude belt $\varphi_{\min} ... 70^{o}$ for secure ($\tau_{\max} / N$) and optimum regular (τ_N^*) constellations versus $\varphi_{\min}$ for the satellite swath Π=2000 km.

The mentioned revisit time benefit values are not maximum possible. At other initial data, they can reach even bigger magnitudes. So, for example, the author has revealed some alternatives when, other things being equal, two-satellite optimum regular system provides lower revisit time value than the four-satellite secure constellation.

4. Analysis of limiting possibilities of regular satellite systems

The analysis presented above was aimed to reveal the fact of actual preferability of regular constellations as compared with secure constellations. In order to provide equal conditions for comparing the possibilities of two classes, an assumption was used that the satellites move along the same trace ("one-route" motion). As for secure constellations, the increase of the number of routes cannot be used to reduce coverage periodicity, but for regular constellations this factor is an additional source for enhancing the orbital structure quality. Regular constellations are in this case difficult to be analyzed due to the lack of analogs.

Therewith, the evaluation of limiting possibilities of regular constellations has another, more important, aspect. As it appears from the regular constellation's definition, their extreme modifications cannot pretend on being absolutely optimal because all regular systems meet only the condition of local optimality. With this, the question arises how close are such modifications to absolute optimum.

So, it is expediently to compare three values of periodicity: τ_G, τ_R, τ_A, corresponding to the best orbital structures, within the following areas:

1) area *G* of secure constellations (τ_G);

2) area *R* of regular constellations (τ_R);

3) area *A* of arbitrary constellations (τ_A).

In the first case, optimization is provided by the method described in [28, 31, 37], in the second and third cases the methods presented in the framework of the route theory are used [41].

It is clear that $G \subset R \subset A$. Thus, in general case τ_G, τ_R, τ_A can not to coincide. At the same time, particular cases when τ_G and τ_R, and τ_R and τ_A or even all three values coincide cannot be excluded. All these situations are really available for τ_G, τ_R, τ_A at various initial data:

$$\tau_G \le \tau_R \le \tau_A. \qquad (8)$$

Therewith, in order to determine significance of regular constellations in the practice of ballistic design of the systems of periodical coverage it is desirable to evaluate maximal distinctions between the values of τ_G and τ_R, τ_R and τ_A.

This evaluation was not so detailed as the research of the regular constellation extreme modifications presented above owing to high labour input needed to find optimal orbital structure. As a result, a series of 500 versions of initial data corresponding with one-day and two-day orbits at N=3 satellites in the system, was analyzed (version with N=2 was not considered because an extremal two-satellite regular system is always absolutely optimal one). Present research led to the following conclusion.

It proved to be that the advantage of extremal versions of regular orbital structures over the guaranteed ones is characterized as follows:

$$1:3{,}6 \le \tau_R : \tau_G \le 1:1, \qquad (9)$$

and the analogous advantage of absolutely optimal orbital structures over the regular ones is characterized as follows:

$$1:1{,}2 \le \tau_G : \tau_A \le 1:1. \qquad (10)$$

It is necessary to add that the distinctions in τ_G and τ_R values took place in the great majorities of cases. It would not be correct to give some magnitude because the coincidence of these two values can be to a great extent foreseen a priori[1]). Lack of coincidences of τ_R and τ_A occurred very seldom (in about 5% of cases), and sometimes they could be eliminated by the accuracy rise of the extremal system calculation.

5. Parameters characterizing periodic coverage geometry and their functional interrelation

Revisit time calculated under the assumption that the satellite swath value is fixed is a traditional criterion of the satellite constellation quality for periodic coverage. But in many important applications the value of revisit time could be a priori given on the basis of specific requirements to the satellite constellation. There is a chance in this case to reduce additionally the observations cost in terms of minimized satellite swath. This problem is discussed below.

Let's introduce the parameters characterizing the geometry of the Earth surface coverage by the satellite swath at periodical coverage, and obtain the interrelations of these parameters.

It will be assumed that the observation region is the Earth latitudinal belt R bounded by parallels R_φ in latitudes $\varphi = \varphi_{min}$ and $\varphi = \varphi_{max}$:

$$R = \bigcup_{\varphi} R_\varphi,\ \varphi \in [\varphi_{min}, \varphi_{max}]. \qquad (15)$$

With allowance for (1), the restriction is applied on the satellite orbit inclination i:

$$i_0 + \beta > \varphi_m,\ i_0 = min(i, \pi - i),$$
$$\varphi_m = max(|\varphi_{min}|, |\varphi_{max}|). \qquad (16)$$

Here β is the angular width of the satellite swath corresponding to the linear swath width $\Pi = 2R_E\beta$, R_E is the Earth mean radius.

Let's consider the coverage of fixed parallel R_φ in latitude φ. In case of $|\varphi| < i_0$ every satellite crosses twice within one circuit the parallel on latitude φ: on the circuit ascending part in point A_j and on its descending part in point D_j (Fig. 5,a). Points A_j and D_j will be called

[1] In fact, it is not difficult to explain theoretically the coincidences of τ_G and τ_R: they have a tendency to occur in situations being trivial enough, when the satellite swath width corresponds to small repetition factors of the area under observation by a single satellite within the period of the trace repetition, and they always occur at a single coverage. In the mentioned cases, when the coverages of single satellites are not great in number and simple in structure, the optimum has to be reached on the multitude of guaranteed systems.

ascending and descending latitudinal nodes. Over a period of T_{TR} the satellite will cross a fixed parallel R_{φ} in m ascending nodes A_j and in m descending nodes D_j. Either of the two systems of points $A = \{A_j\}$, $D = \{D_j\}$, $j = 1,2,3,\ldots,m$ is uniform, and distance between neighbouring points in these systems (of points) is $\Delta L = 2 \cdot \pi / m$ (internodal distance – the distance between neighbouring points of intersection with equator of the one satellite trace ascending parts within the repetition period T_{TR} of this trace).

Union $A \cup D$ in general case yields an irregular set of points characterized by angular shifts δ_R and δ_L (right and left shifts of the systems of points A and D – see Fig. 5,a). In substance, shifts δ_R and δ_L are minimal angles to turn round a system of points A towards increasing (δ_R) and decreasing (δ_L) longitudes of the nodes till it coincides with a system D of points. Obviously, $\delta_R + \delta_L = \Delta L$.

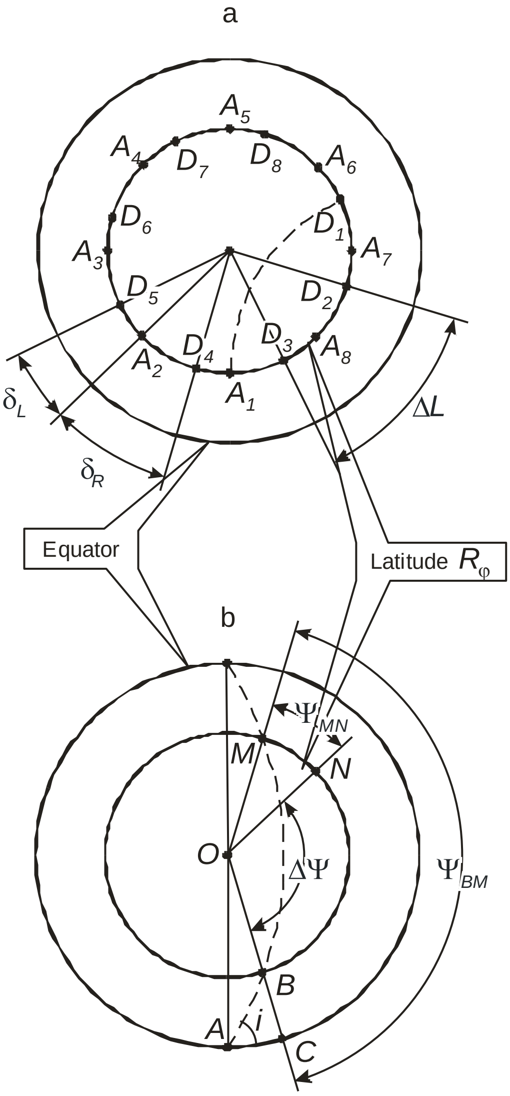

Fig.5: Relative displacement on the Earth parallel of the ascending and descending latitudinal nodes (view from the pole)

To evaluate shifts δ_R and δ_L, it is convenient to consider angle of turn $\Delta\psi$ (turn of the systems of points A and D) that is the difference of longitudes of latitudinal nodes A_k and D_k, belonging to any kth-circuit of one satellite. It follows from Fig. 5,b that $\Delta\psi = \psi_{BM} - \psi_{MN}$, where ψ_{BM} is the difference of longitudes of points B and M of the orbit crossing with parallel R_φ on non-rotating Earth; ψ_{MN} – rotational displacement (angle of rotation) of the Earth during the time period of the satellite orbital flight from point B to point M. From the analysis of spherical triangle ABC (Fig. 5,b) the following expression can be obtained for angle of turn $\Delta\psi$:

$$\Delta\psi = \pi \cdot \left(1 - \frac{n}{m}\right) - 2 \cdot \left[arcsin\left(\frac{tg\varphi}{tgi}\right) - - \frac{n}{m} \cdot arcsin\left(\frac{sin\varphi}{sini}\right)\right]. \qquad (17)$$

Suppose that G is a selection function of the residual of division, the expressions for δ_R and δ_L will be: $\delta_R = G(\Delta\psi/\Delta L)$, $\delta_L = \Delta L - \delta_R$.

As it is seen from the expressions given above, shifts δ_R, δ_L are functions of m, n, i, φ. Alteration of these parameters depending on latitude φ is of oscillatory nature which is related to specific values of m, n, i. Dependencies $\delta_R(\varphi)$ and $\delta_L(\varphi)$ are shown for example in Fig. 6 for $m=1$, $n=16$, $i=65^o$ (orbit height $H = 226$ km).

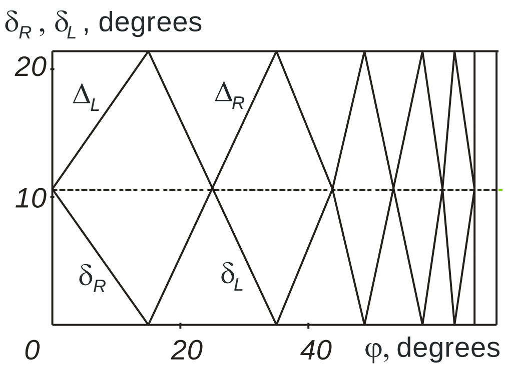

Fig. 6: Dependencies between right and left shifts and the parallel latitude *(m=16, n=1, i=65^0)*

Let R_Π be a satellite swath on the rotating Earth surface realized for angular width β of the satellite swath. Let's call a length α_R (α_L) of the circuit part R_φ covered by R_Π from its axial line, i.e. from the satellite trace, to its right (left) border, as right-side (left-side) lock-on of swath R_Π on parallel R_ϕ. Locks-on in ascending and descending latitudinal nodes will be distinguished by identifying them in case of need, by upper and lower index marks "A" and "D", respectively: α_R^A, α_L^A, α_R^D, α_L^D.

It can be shown that owing to the symmetry of parallel R_φ coverage by swath R_Π, the equalities $\alpha_R \equiv \alpha_R^A = \alpha_L^D$, $\alpha_L \equiv \alpha_L^A = \alpha_R^D$ are right. Sum $\alpha = \alpha_R + \alpha_L$ will be called lock-on of swath R_Π on parallel R_φ.

The locks-on α, α_R, α_L are functions of the swath width Π, latitude φ of covered parallel R_φ, as well as inclination i and repetition factor m/n of the satellite geosynchronous orbit. These locks-on are monotonous functions of the swath width $\mathit{\Pi}$ at fixed parameters m, n, i, φ:

$$\alpha = \alpha(\mathit{\Pi}), \alpha_R = \alpha_R(\mathit{\Pi}), \ \alpha_L = \alpha_L(\mathit{\Pi}). \qquad (18)$$

Geometric characteristics considered above are referred to the parallel coverage by a single satellite swath. As for the system of several satellites, total coverage of the parallel by their swaths is formed by superposable coverages provided by separate satellites. Relative position of the coverages of different satellites on any parallel R_φ is characterized unambiguously by specified longitudes $L_j, j = 1(1)N$ of the system satellites traces [41].

By analogy with the Earth continuous observation, a notion of the Earth surface coverage repetition factor l in the problem of periodical coverage is introduced. Magnitude l will be used to characterize the coverage of point r or region R of the Earth (in particular case, of the parallel or latitudinal belt) created by the swaths of one or several (all) satellites of the system. It will be understood that l-fold coverage of point r is provided, if during the time interval T_{TR} between the satellites (satellite) traces repetition this point l times finds itself in the instantaneous zones of the satellites (satellite) coverage, and l-fold coverage of region R is provided, if during the time T_{TR} not less than l-fold coverage of every point $r \in R$ is provided. If point r never comes upon the instantaneous coverage zones of the satellites (satellite) during the whole period T_{TR}, it should be said that the coverage uniformity of r is not provided (or there is no coverage uniformity of r). If there is even one such point among points $r \in R$, it will be said that the coverage uniformity of R is not provided (there is no coverage uniformity of region R).

It should be noted that according to the definition of l-fold coverage, the minimal swath width $\Pi_l[R]$ providing that coverage for latitudinal belt R is connected to the values $\mathit{\Pi}_l\lfloor R_\varphi \rfloor$ of this parameter for separate parallels $R_\varphi \in R$ by the formula:

$$\Pi_1[R] = \max \Pi_1\lfloor R_\varphi \rfloor, \quad R_\varphi \in R. \qquad (19)$$

So, in order to solve the problem of the latitudinal belt multiple coverage it is enough to do it for one parallel $R_\varphi \in R$. This fact is used from now on. At that, for simplicity sake, a parameter Π_l. is used instead of $\mathit{\Pi}_l\lfloor R_\varphi \rfloor$.

6. Problem of satellite design for periodic coverage with minimal satellite swath: general solution and some known particular cases

With allowance for introduced designations, a general solution of the problem of multiple (l-fold) coverage of the parallel by a single satellite swath for odd ($l = 2k-1$) and even ($l = 2k$) cases ($k = 1, 2, 3, \ldots$) is as follows [33].

$$\Pi_l = \Pi_l[R\ ] = = \max\left\{ \min\left[\alpha_R^{-1}\left(\frac{(k-1)\cdot\Delta L + \delta_R}{2}\right), \quad \alpha_L^{-1}\left(\frac{k\cdot\Delta L + \delta_L}{2}\right)\right], \min\left[\alpha_L^{-1}\left(\frac{(k-1)\cdot\Delta L + \delta_L}{2}\right), \quad \alpha_R^{-1}\left(\frac{k\cdot\Delta L + \delta_R}{2}\right)\right]\right\}, \quad (20)$$

$$\Pi_l = \Pi_l[R_\varphi] = \alpha^{-1}(k\cdot\Delta L), \quad l = 2k\ . \quad (21)$$

Functions being inverse to functions (18) are designated here as $\alpha_R^{-1},\ \alpha_L^{-1},\ \alpha^{-1}$. Expressions (20) and (21) allow to define multiple swaths (i.e. minimal swaths providing coverage of prescribed repetition factor l) for any given factors m/n, values of geosynchronous orbits inclinations i and latitude φ.

It follows from the analysis of expression (21) for even coverage that a required swath width Π_l is in that case maximal for parallels R_φ, located in the lesser (in absolute value) latitude φ. Actually, a linear size of the arc of constant angular value ΔL decreasing with increasing $|\varphi|$ on each of analysed parallels, is of determining influence on Π_l, $l = 2k$.

The known method widely used in the engineering practice – to design the satellite systems on the basis of assigning the Earth surface non-miss observation swath equal in magnitude to inter-circuit distance ΔL_{IC} (the distance between ascending nodes of the satellite orbit on the equator (latitude) on two subsequent circuits) in a minimal latitude of the observation region, can serve as an application example of the abovementioned property of relation $\Pi_l(\varphi)$ at $l = 2, 4, 6, \ldots$:

$$\Pi_\varphi = \Delta L_{IC} = n\cdot\Delta L. \quad (22)$$

Π_φ characterizes the swath width measured not perpendicular to the satellite orbit plane (as it was thought before, see [41]) but along the equator (the parallel in a minimal observation latitude). It is easy to see that swath chosen in such a way is the swath of even $2n$-fold coverage because internodal distance ΔL is here always equal to $1/n$ part of inter-circuit distance ΔL_{IC}. In a particular case of round-the-clock ($n=1$) orbit it will be a 2-fold ($l=2$) swath $(\Delta L_{IC} \equiv \Delta L)$.

Another way to choose a minimum needed swath is the method [44] including the swath representation reasoning from repetition factor m/n of the used geosynchronous orbit of the satellite in accordance with expression

$$\Pi_{\varphi} = \Delta L_{IC}/n = \Delta L = \frac{2\pi}{m}. \quad (23)$$

Formula (23) is a generalization of (22), and it fully defines the 2-fold swath width in compliance with the terminology introduced above in the paper.

It can be shown that the observation periodicity τ of the Earth regions with 2-fold swath is within the limits $\tau \in (T_{TR}/2, T_{TR})$. So, if the 2-fold swaths are used, the chosen swath can turn out to be unduly high, and periodicity – needlessly low, when it is enough to solve the problem with periodicity $\tau = T_{TR}$. The right way in this situation is the use of swaths with the odd repetition factor l and first of all single swaths, i.e. the least swaths providing non-miss observation of the region with periodicity $\tau \le T_{TR}$.

Application of formulas (22) and (23) is based on that the function $\Pi_l(\varphi)$ decreases monotonously with growing φ for any even repetition factor l. Width Π_{φ} of the swath can be thus prescribed in minimal latitude of the observation region (owing to the symmetry of the Earth coverage in the Southern and Northern hemispheres it is here and after supposed that the latitude changes within the interval $0 \le \varphi < 90^o$). Relationship $\Pi_l(\varphi)$ for the odd repetition factor l has qualitatively another character, with sudden changes, because in this case a relative disposition of the parallel ascending and descending nodes has additional influence upon Π_l value. Physical essence of such an influence is illustrated in Fig. 3 by means of a single-swath example. The parallel R_{φ} coverage diagrams are given in Fig. 3 for some ranges of right shift δ_R variation. Critical points r^* of a single swath, i.e. the mating points of the swaths providing the coverage continuity, are shown, too. Every range of δ_R alteration corresponds to single ($l=1$) coverage characterized by angular swath width β_1.

All designations in Fig. 3 are related to a particular case of expression (20) – swath width $\beta_1 = \Pi_1/(2R_E)$ of single ($l = 1$) coverage in angular measurement:

$$\beta_1 = \max\left\lfloor \min\left(\beta^{(1)}, \beta^{(4)}\right), \min\left(\beta^{(2)}, \beta^{(3)}\right)\right\rfloor, \qquad (24)$$

where

$$\beta^{(1)} = \alpha_R^{-1}(\delta_R/2),$$

$$\beta^{(2)} = \alpha_L^{-1}(\delta_L/2),$$

$$\beta^{(3)} = \alpha_R^{-1}((\Delta L + \delta_R)/2),$$

$$\beta^{(4)} = \alpha_L^{-1}((\Delta L + \delta_L)/2).$$

This implies that in general case function $\Pi_l(\varphi)$, is not monotonous at single ($l = 1$) coverage (and also at any other coverage with the odd repetition factor l, as can be shown), and can have several local extremes. Local maxima $\Pi_l(\varphi)$ are thus reached in latitudes where a pair of ascending and a pair of descending latitudinal nodes mate simultaneously in critical points r^*, (that is case "c" in Fig. 3, and right shift is here $\delta_R = \delta'_R$). Local minima are reached in latitudes where there are not one (as in all other cases) but two "equivalent" critical points r^* of mating ascending and descending nodes (that is case "e", $\delta_R = \delta_R''$) on each interval of length ΔL. It is interesting to note that shift $\delta_R = \delta'_R$ at $\alpha_L = \alpha_R$ corresponds to full coincidence of ascending and descending nodes, and shift $\delta_R = \delta_R''$ corresponds to a uniform distribution of all latitudinal nodes along the parallel. Diagrams in Fig. 7 are represented for $\alpha_L \le \alpha_R$, and β_1 possesses one of three values $\beta^{(1)}, \beta^{(2)}, \beta^{(3)}$. It can be shown that at $\alpha_R \le \alpha_L$ the picture of the parallel coverage will be analogous, and instead of $\beta^{(3)}$ will "work" $\beta^{(4)}$ in formula (24).

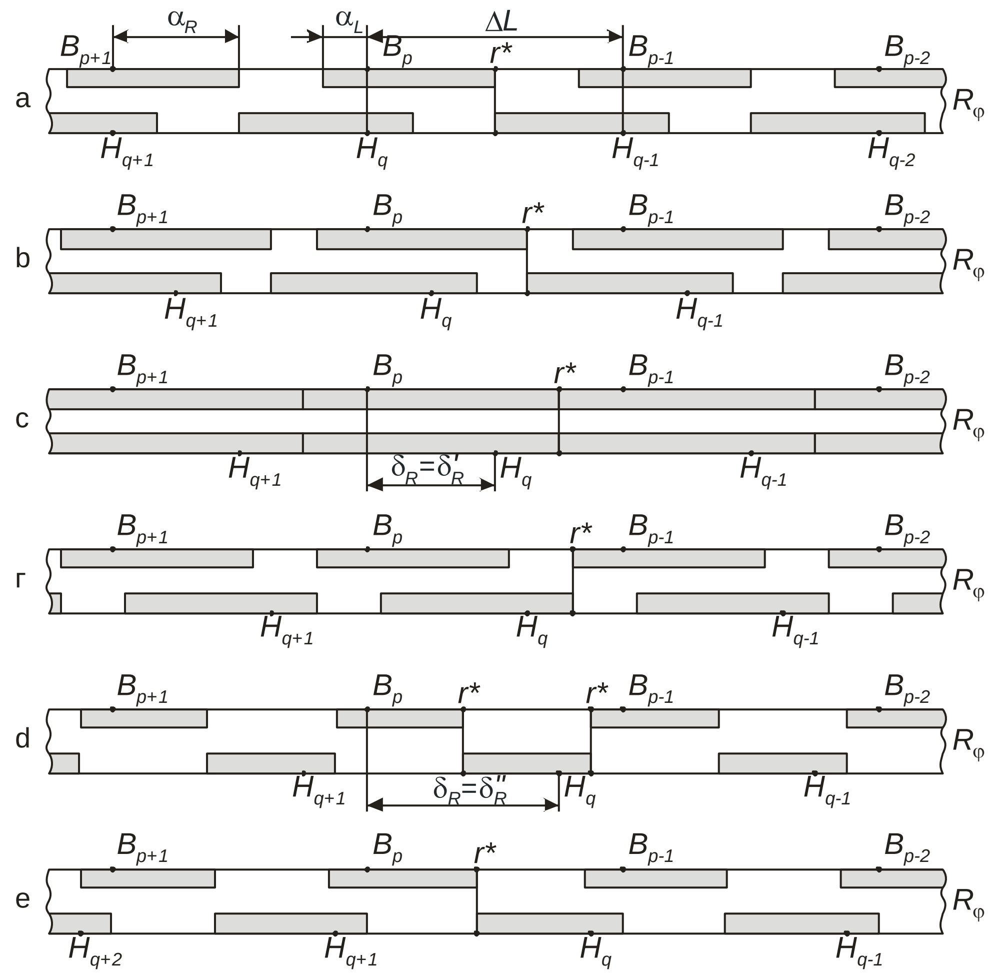

Fig. 7: Typical diagrams for single coverage of the parallel depending on relative disposition of the systems of ascending and descending nodes:

a) $\delta_R = 0\ (\delta_R = \Delta L),\ \beta_1 = \beta^{(3)}\ (\beta_1 = \beta^{(1)})$; b) $0 < \delta_R < \delta_R^{'},\ \beta_1 = \beta^{(3)}$;
c) $\delta_R = \delta_R^{'},\ \beta_1 = \beta^{(3)} = \beta^{(2)}$; d) $\delta_R^{'} < \delta_R < \delta_R^{''},\ \beta_1 = \beta^{(2)}$;
e) $\delta_R = \delta_R^{''},\ \beta_1 = \beta^{(2)} = \beta^{(1)}$

It follows from above that the width of the parallel non-miss observation can be reduced by means of modification of relative position of ascending and descending latitudinal nodes. It is easy to make sure that in such a way the swath width of non-miss observation of large latitudinal belts can be decreased. Really, it can be obtained from the analysis of the plots $\Pi_l(\varphi)$ in Fig. 8 that maximal swath for the odd ($l = 2k - 1$)-fold coverage can be less than analogous swath of even ($l = 2k$)-fold repetition for some latitudinal belts ($k = 1, 2, 3, \ldots$). For example, such latitude interval is $10^o \ldots 65^o$ for $m = 15,\ n = 1,\ i = 65^o$ (Fig. 8,b).

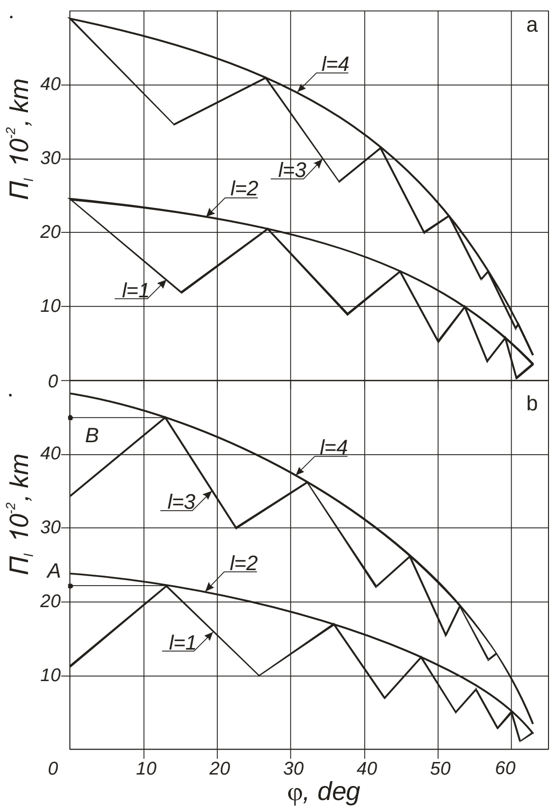

Fig. 8. Plots of the swath width Π_l for *l*-fold coverage of the parallel against its latitude φ for different values $l = \{1,2,3,4\}$ and parameters of the satellite orbit:
a – *m=15, n=1, i=65*^{0}
b – m=16, n=1, i=65^0

Several special methods to design the satellite constellations with minimum swath width for non-miss coverage of the Earth latitudinal belts, are considered below. All these methods are based on the consideration of relative dispositions of the ascending and descending nodes on the parallels.

7. Method of satellite design for latitude belt coverage with minimal swath on basis of optimization of orbit altitude and inclination [45]

Alteration of the ascending and descending nodes relative position will be considered over the range $\varphi \in [0, \varphi_i]$, $\varphi_i = \min(i, \pi - i)$ of latitudes covered by the swath of a single satellite moving in the orbit with inclination i. For that, the values of right shift δ_R will be calculated as a function of latitude φ on the equator $(\varphi = 0)$ and on the maximal latitude $\varphi = \varphi_i$.

From (3) at $\varphi = 0$, angle of turn is $\Delta\psi = \pi[(m-n)/n]$. With allowance for that, right shift δ_R is as follows:

$$\frac{\Delta\psi}{\Delta L} = \frac{\pi(m-n)}{m \cdot \frac{2\pi}{m}} = \frac{m-n}{2},$$

$$\Delta\psi = \frac{m-n}{2} \cdot \Delta L,$$

$$\delta_R = \Delta\psi - \Delta L \cdot E\left(\frac{\Delta\psi}{\Delta L}\right) =$$

$$= \frac{m-n}{2} \cdot \Delta L - \Delta L \cdot E\left(\frac{m-n}{2}\right) =$$

$$= \Delta L \cdot \left[\frac{m-n}{2} - E\left(\frac{m-n}{2}\right)\right]. \qquad (25)$$

It results from (25) that if difference ($m-n$) is an even number ($m-n=2k$, $k=1,2,3,\ldots$), than the systems of latitudinal nodes A and D on the equator coincide:

$$\delta_R = \Delta L \cdot \left[\frac{2k}{2} - E\left(\frac{2k}{2}\right)\right] = \qquad (26)$$
$$= \Delta L \cdot [k - E(k)] = 0.$$

If difference ($m-n$) is an odd number ($m-n=2k-1$, $k=1,2,3,\ldots$), than all ascending and descending latitudinal nodes are located on the equator uniformly:

$$\delta_R = \Delta L \cdot \left[\frac{2k+1}{2} - E\left(\frac{2k+1}{2}\right)\right] == \Delta L \cdot \left[k + \frac{1}{2} - E\left(k + \frac{1}{2}\right)\right] = \frac{\Delta L}{2}. \qquad (27)$$

At $\varphi = \varphi_i = i < \pi/2$, angle of turn $\Delta\psi = 0,$ wherefrom at any m, n the right shift $\delta_R = 0.$ Similarly, it will be obtained at $\varphi = \varphi_i = \pi - i, i > \pi/2$ that $\delta_R = \Delta L.$ So, in a maximal latitude $\varphi = \varphi_i$ the ascending and descending latitudinal nodes always coincide.

The obtained conditions define the right shift values $\delta_R(\varphi)$ on the latitude range $\varphi \in [0, \varphi_i]$ borders. The right shift δ_R alteration as well as the left shift $\delta_L = \Delta L - \delta_L$ alteration within this range are in general case of oscillatory character depending on specific values m, n, i (see Fig. 6). The non-miss coverage swath of any parallel will be determined by maximal distance $max(\delta_R, \delta_L)$ between the latitudinal nodes. That is why the most economical coverage of the equator is realized at moving of the satellite on geosynchronous orbits with the odd difference ($m-n$) providing a uniform distribution of the nodes on the equator. In this case, for a non-miss coverage of the equator

$$\Pi_\varphi = \Delta L / 2 = \pi / 2. \quad (28)$$

In order to provide swath (28) non-miss coverage of not only the equator but also any other parallel $R_\varphi, \varphi \in [0, \varphi_i]$, it is necessary vector $\bar{v}$ of the subsatellite point displacement rate in any equatorial node be in the meridian plane. Displacement rate $\bar{v}$ of the subsatellite point relative to the Earth is a resulting vector of absolute velocity $\bar{v}_A$ due to the satellite motion along the orbit and of transportation velocity $\bar{v}_E$, caused by the Earth rotation (Fig. 9). Inclination i^* conditioning the velocity $\bar{v}$ direction being strictly at a tangent to the meridian is defined by the expression

$$i^* = arccos(v_E / v_A), \quad (29)$$

where v_A, v_E are the values of respective vectors $\bar{v}_A, \bar{v}_E$, defined as

$$v_A = \frac{2\pi R_E}{T_{DR}}, \quad v_E = \frac{2\pi R_E}{T_{EF}}, \quad (30)$$

It will be obtained after substitution of (16) into (15) and transformation:

$$i^* = arccos(n/m). \quad (31)$$

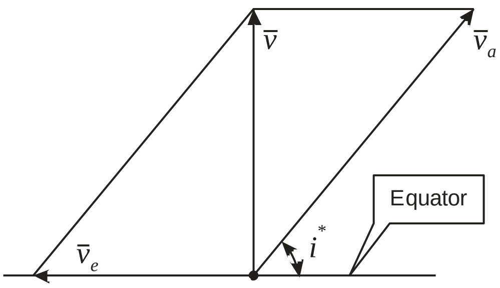

Fig. 9. To the evaluation of inclination (31)

Relationship (31) for the range of altitude 200...4000 km is represented in Fig. 10. It is supposed that the plot is not an uninterrupted line but consists of discrete points corresponding to rational numbers m/n on the abscissa axis.

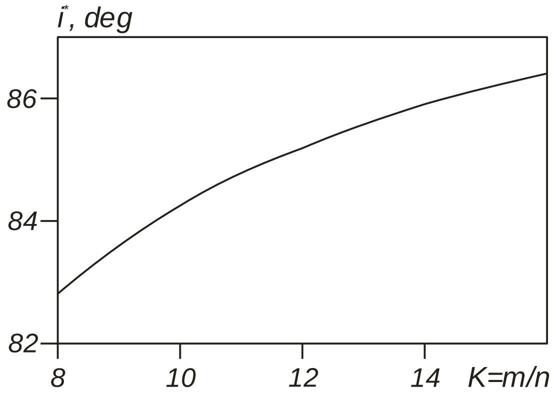

Fig. 10. Dependence of inclination (31) on the geosynchronous orbit repetition factor

Thus, the satellite moving along geosynchronous orbit with repetition factor m/n, odd difference $m-n$, and having inclination (31) provides the most efficient coverage of the Earth surface – it is enough to have for that the swath of width (28). The physical essence of the resulting coverage efficiency can be illustrated in Fig. 11 similar to Fig. 8 and differing by that the inclination defined by formula (31) is used in the figure. It can be seen from these two figures comparison that spasmodic graphs $\Pi_l(\varphi)$ represented in Fig. 8 for $l=1$ and $i=65^o$, become straight in Fig. 11 at inclination (31). When difference of the orbit parameters m and n is an odd number, then maximal value $\Pi_l(\varphi=0)$ for repetition factor $l=1$ becomes sufficiently less than the analogous value for repetition factor $l=2$. It should be noted that the same conclusion would hold with respect to maximal value of $\mathit{\Pi}_l(\varphi=0)$, and also at the comparison of repetition factors of higher order - the case $l=3$ and $l=4$ is analysed in detail below.

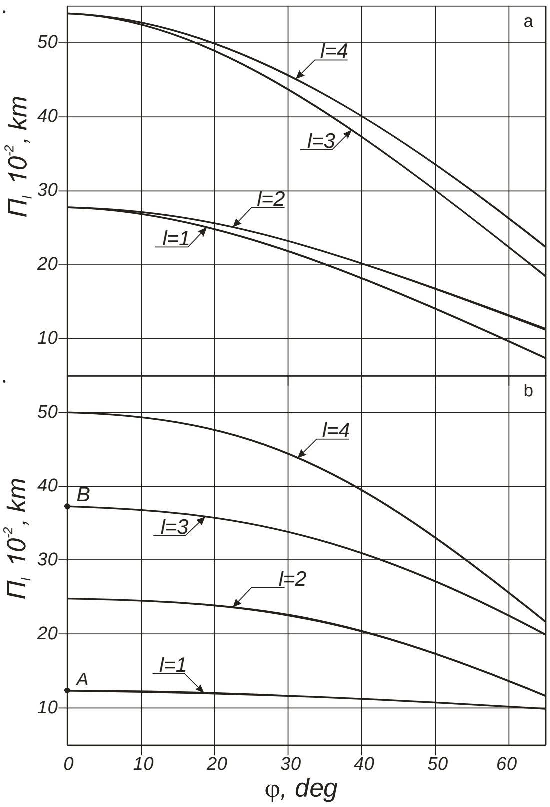

Fig. 11. Plots of the swath width Π_l for l -fold coverage of the parallel against its latitude φ for different values $l = \{1,2,3,4\}$ and parameters of the satellite orbit:
a – *m=15, n=1; i=arccos(1/15)≈86,2*^{0}
b – *m=16, n=1; i=arccos(1/16)≈86,4*^{0}

To compare the efficiency of this method of the orbit formation with traditional method described by expression (23), the observation of the latitude belt $0..70^o$ by means of a single satellite moving on geosynchronous orbit with repetition factor $m/n = 16/1$ (with the odd difference $m - n = 16 - 1$), will be considered. Taking after traditional approach, inclination $i = 70^o$, can be proposed for the region observation, and the swath width is chosen in accordance with (23) for a double swath. The method under consideration supposes the use of inclination $i = \arccos(16/1) \approx 86{,}4^o$, swath corresponds to the single one. The dependencies between $\Pi_{l max} = \max_{\varphi} \Pi_l(\varphi), \varphi \in [0, \varphi_i]$ and inclination i for traditional ($l = 2$) and proposed ($l = 1$) methods allowing to choose the swath Π are presented in Fig. 12. It is seen that in the first case Π is 2410 km (point A, $i = 70^o$), and in the second case – 1250 km (point B, $i = 86^o{,}4$). So, the proposed method permits to reduce the necessary swath of the satellite by a factor of $2410 / 1250 \approx 1{,}9$.

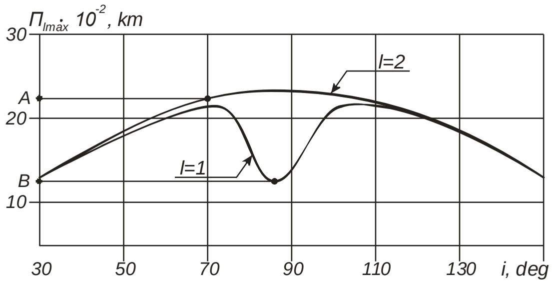

Fig. 12: Relationships between $\Pi_{l\,max}$ value and inclination for traditional ($l=2$) and proposed ($l=1$) methods to choose the swath width

Observation by one satellite connected with large time breaks was considered above. The placement of a big number of satellites on one circuit under the same conditions allows to apply directly the proposed method for designing the satellite systems for observation with small periodicities.

8. Method of satellite design for low-cost multiple coverage using optimal altitude and inclination [46]

The coverage repetition increase is one of the ways to rise the observation periodicity. So, it is important to find the methods to minimise the swaths for multiple coverages. Let's appeal with this end in view to Fig. 9 showing the arrangement of ascending and descending latitudinal nodes on the equator at the odd difference $m-n$ of parameters m, n of the geosynchronous orbit. It is seen in this figure that one latitudinal node (ascending or descending) occurs in the satellite swath of width π/m within the period T_{TR} of the trace repetition, two nodes (ascending and descending) occur in the swath of width $2\pi/m$, three nodes – in the swath of width $3\pi/m$, etc. The number of the nodes falling into the swath of nodes corresponds actually to the number of observation sessions of the arbitrary equator point during time T_{TR}, i.e. to the repetition factor l of the equator coverage. The choice of inclination according to (31) provides the repetition factor value being not less than that for the equator.

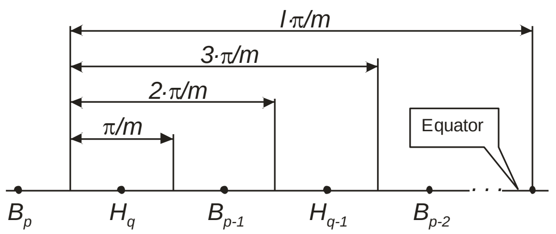

Fig. 13: Relative position of ascending and descending equatorial nodes at an odd difference ($m-n$) of parameters m, n of the satellite geosynchronous orbit

Thus, the conclusion can be made that in order to obtain the l-fold observation of the Earth surface by the satellite during the period of its trace repetition it is enough to provide under prescribed conditions the swath width of

$$\Pi_{\varphi} = \pi l / m. \qquad (32)$$

The particular case of expression (32) at $l = 1$ results in (14). At even repetition factors $l = 2k, k = 1, 2, \ldots$ expression (32) is reduced to (23). The cases of odd repetition factor numbers $l = 2k + 1, k = 1, 2, \ldots$ give new ways to provide the Earth coverage. Let's analyse them.

Suppose the need to provide a single satellite observation of the Earth latitudinal belt $0 \ldots 65^{o}$ with periodicity not more than 24 hours and, if possible, lesser width of the satellite swath. Have also the restrictions for altitude H and inclination i of the satellite orbit defined by the following ranges of these parameters alterations often used in the practice: $H = 600..800$ km, $i = 65^{o}..90^{o}$. Fig. 14 illustrates the dependencies $\Pi_{lmax}(i)$, similar to those in Fig. 8 but calculated for various repetition factors l of the coverage and for different parameters m/n, corresponding to the ($n = 1$)-, ($n = 2$)- and ($n = 3$)- orbits from the altitude range 500...900 km.

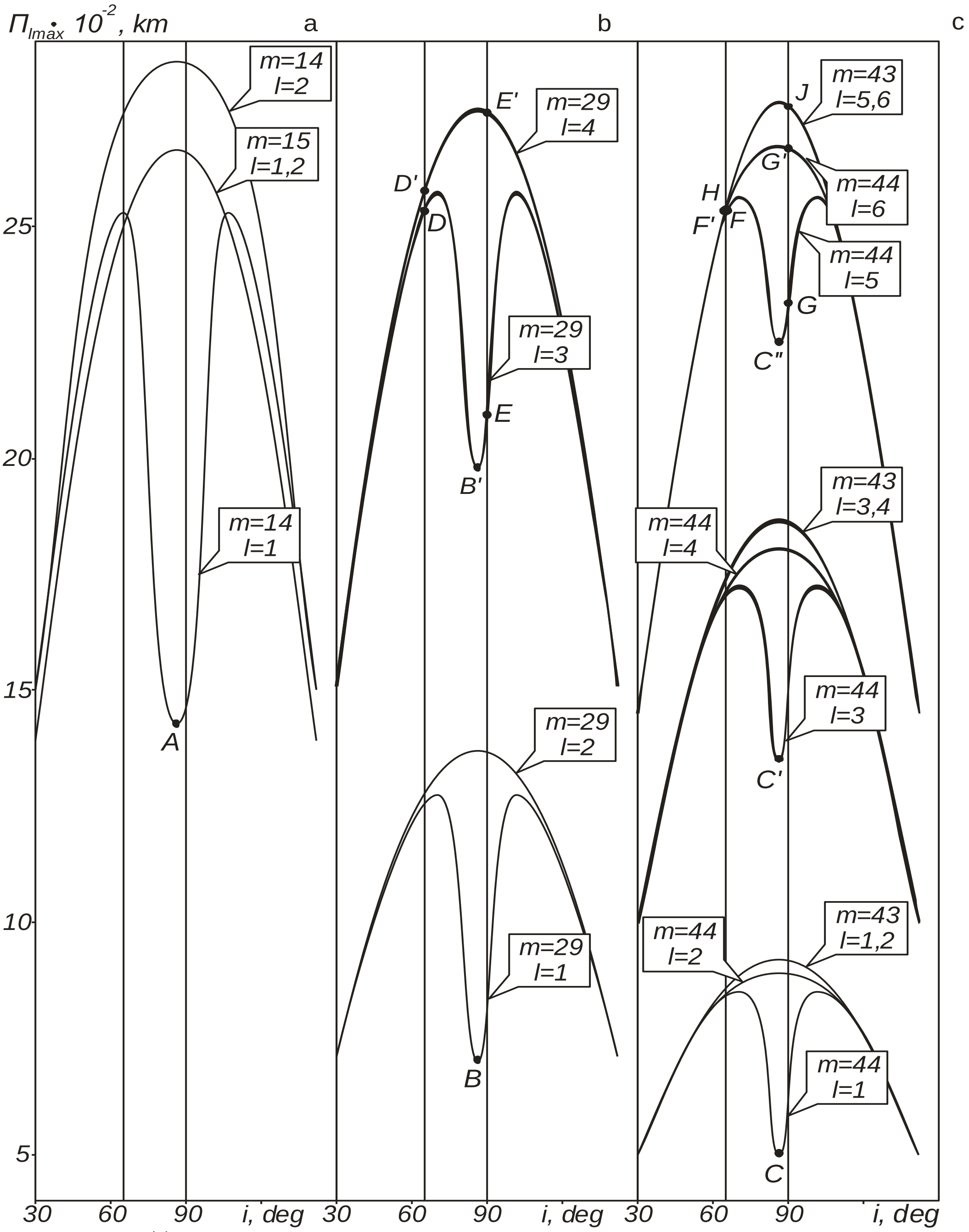

Fig. 14. Plots $\Pi_{l\max}(i)$, at different repetition factors *l* of the coverage of geosynchronous orbits with altitudes 500…900 km during one - (a – n=1), two- (b – n=2) or three (c – n=3) efficient astronomical days

The relationships between altitudes and inclinations of the analysed orbits are shown in Fig. 11.

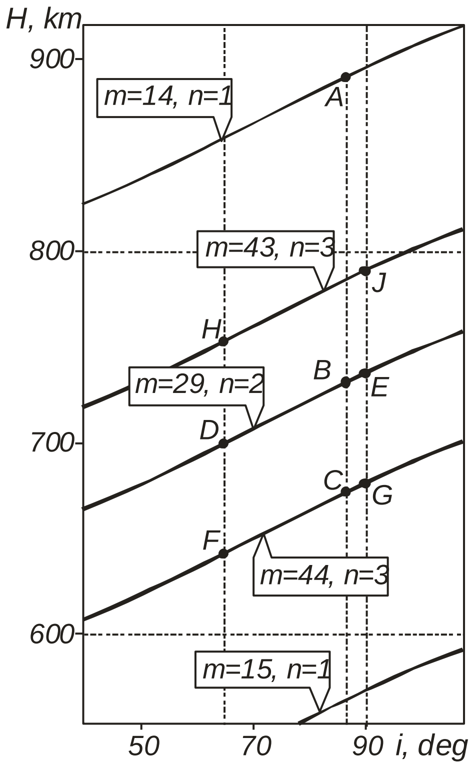

Fig. 15. Graph of altitudes for geosynchronous orbits within the altitude range 500...900 km of one-, two- and three astronomical days, versus inclination

It is clear that points A, B, C in Fig. 14 are related to the realisation of the method presented in section 7. Only one of these point meets at that the requirements for periodicity (periodicity in point A $\tau \approx 1$ astronomical day, in point B – $\tau \approx 2$ astronomical days, in point C – $\tau \approx 3$ astronomical days). However, this point proved to be out off the acceptable range of altitude, namely 600...800 km. Only the realisations of a single-satellite observation corresponding to parts DE, $D'E'$, FG, $F'G'$, HJ in Fig. 14 meets fully all the demands in altitude and periodicity. Each of these parts corresponds to analogous (designations without primed symbols) parts DE, FG, HJ in Fig. 15.

Parts $D'E'$, $F'G'$, HJ in Fig. 14 conform to the methods of the swath width choice considered above. The lowest swath width under the prescribed restricting conditions is for these methods ≈ 2500 km. As it can be seen the gain in the swath size due to the use of the method under consideration is $(2500-2000)\cdot 100\% / 2500 = 20\%.$ The example of the realisation method being proposed can be easily modified in the context of other initial data including multi-satellite systems.

9. Method of multi-satellite network design for low-cost multiple coverage using optimal altitude and inclination [47]

In use of the orbits with optimal altitude and inclination in accordance with section 7, a possibility appears to lower the needed swath for latitudinal belt $0..\varphi_i$ still more while applying additional $N-1$ satellites in the orbital group. For instance, if the traces of all N satellites within internodal interval ΔL (in particular case, within inter-circuit interval $\Delta L_{IC} = n \cdot \Delta L$), the next generalisation can be obtained as N-route satellite system. It can be shown that in this case the choice of the repletion factor will be realised under condition $N(m-n) = 2k-1$ ($k = 1, 2, 3, \dots$), and the swath will be defined by expression $\Pi_\varphi = \pi / (mN)$, generalising formula (14). In the event, such placement of the satellite traces does not always give the most economical advantage. The more efficient method for the satellite traces disposition in order of the swath minimisation will be examined below.

Traditional methods of the satellites N traces relative placement are reduced to their uniform distribution (with longitudinal spacing ΔL_{TR}) within internodal interval ΔL (in particular case within inter-circuit interval ΔL_{IC}), and to the choice of the satellite swath width Π_φ equal to

$$\Pi_\varphi = \Delta L_{TR} = \Delta L / N = 2\pi / mN. \qquad (33)$$

Under the conditions of optimal choice of the altitude and inclination according to the above, when each interval between adjacent like nodes on the equator is divided in two equal parts by the node of another type, the placement of another satellite with longitudinal spacing (33) will not lead to the needed swath width reduction because all latitudinal nodes of the second satellite will in this case coincide accurately with those of the first satellite.

It can be shown that the most efficient use of the locks-on of all available N satellites swaths with the aim to their value in need consists in the providing of uniform distribution along the equator of the set of latitudinal nodes of all N satellites that allows to reduce the observation swath proportionally to number N. Such uniform displacement is possible to be gained by placing the N satellites traces with longitudinal shift (Fig. 16)

$$\Delta L_{TR} = \begin{cases} 2\pi / mN, & N = 2k+1, \\ & k = 1,2,3,\ldots \\ \pi / mN, & N = 2k, \\ & k = 1,2,3,\ldots. \end{cases} \qquad (34)$$

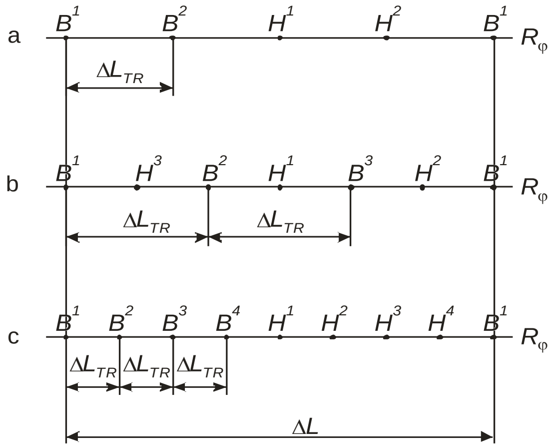

Fig. 16: Optimal relative arrangement of ascending and descending equatorial latitudinal nodes for two (a), three (b), and four satellites

The swath width is obtained by generalization of (32)

$$\Pi_{\phi} = \pi \cdot l / mN. \qquad (35)$$

Comparison of expressions (35) and (32) attests directly the method efficiency in minimization of the swath width.

10. General method of multi-satellite network design for low-cost coverage [48]

Note that $\alpha_R^{-1}(\Delta)$ и $\alpha_L^{-1}(\Delta)$ characterise the satellite swath width (measured along the parallel) necessary for the parallel arc Δ covering accordingly on the right and on the left from the parallel intersection point by the satellite trace. Write symbols $\beta_R(\Delta)$ и $\beta_L(\Delta)$. for them. Expression (10) assumes in the new notations the form:

$$\beta_1 = \max\{\min[\beta_R(\Delta_1), \beta_L(\Delta_2)], \min[\beta_L(\Delta_3), \beta_R(\Delta_4)]\}, \qquad (36)$$

where

$$\Delta_1 = \delta_R / 2,$$

$$\Delta_2 = (\Delta L + \delta_L)/2,$$

$$\Delta_3 = \delta_L / 2,$$

$$\Delta_4 = (\Delta L + \delta_R)/2.$$

Maximal value of (36) obtained for separate parallels in latitudes φ has to be taken to provide the coverage of not a single parallel but latitudinal belt $\varphi_{\min}..\varphi_{\max}$:

$$\Pi_{1\max} = \max_{\phi} \Pi_1(\varphi), \ \varphi \in [\varphi_{\min}, \varphi_{\max}]. \tag{37}$$

Maximum (37) can be reached in any latitude $\varphi = \varphi^* \in [\varphi_{min}, \varphi_{max}]$. One of the parameters $\beta_R(\Delta_1),\ \beta_L(\Delta_2),\ \beta_L(\Delta_3),\ \beta_R(\Delta_4),$ calculated for latitude φ^* will correspond (and be equal) to the width value of the non-miss coverage swath for the whole latitudinal belt. One of the parts $\Delta_1, \Delta_2, \Delta_3$ or Δ_4 (let's denote it χ) related to this maximal value is critical – the largest swath is required to cover it.

Thereby, for decreasing the needed swath of the latitudinall belt $[\varphi_{min}, \varphi_{max}]$ owing to the use of not one but several satellites in proportion to their total number N it seems to be expedient to place the satellite traces with longitudinal spacing.

$$\Pi_{\varphi} = \Delta L_{TR} = 2\chi / N. \tag{38}$$

It is seen from the comparison of (38) with analogous expression (33) and with allowance for $\chi \le \Delta L / 2,$ that the method under consideration provides in general case a more effective coverage of the Earth surface. It can be shown that the gain obtained in the swath width depending on requested parameters H, i of the orbits (as well as of the observation latitudinal belt) varies from the values corresponding to such an optimal request (see section 5) to zero (there is no gain at $\chi = \Delta L / 2$).

11. Некоторые закономерности оптимальных решений

Прикладное значение маршрутной теории оптимизации не ограничивается решением классической задачи периодического обзора. Ее более важная роль состоит в том, что она предлагает новую парадигму мышления при анализе взаимовлияния более широкого набора проектно-баллистических характеристик спутниковых систем периодического обзора. Одним из способов такого анализа является рассмотрение так называемых уровенных поверхностей периодичности обзора – поверхностей в пространстве параметров «периодичность обзора – высота орбит – ширина полос обзора спутников». Действительно, на первый взгляд может показаться, что, ограничиваясь анализом геосинхронных орбит, мы не можем «увидеть», что происходит в пространстве параметров, включающем высоту. Вместе с тем, и здесь предлагаемый подход оказывается крайне полезным. Покажем это.

На рис. 17 показан фрагмент уровенной поверхности периодичности обзора широтного пояса $0...70^o$ для простейшей системы из одного спутника, движущегося по орбите с наклонением $i = 85^o$. Можно показать, что любые попытки получить аналогичную уровенную поверхность для системы из N>1 спутников в рамках лучших известных (гарантированных) орбитальных структур окончатся неудачей в связи с тем, что уровенные линии для различных соседних кратностей m/n (сечений H=const) не будут "стыковаться" между собой. Более того, в этом случае уровенные линии в сколь угодно близких сечениях $H_1 = const$ и $H_2 = const$ могут столь сильно отличаться, что это вызывает определенные сомнения в корректности используемых методов оптимизации (в классе гарантированных орбитальных структур). Именно данное обстоятельство в свое время навело автора на мысль о существовании более лучших структур чем гарантированные, каковыми и явились регулярные СС.

На рис. 18 изображена уровенная поверхность, аналогичная показанной на рис. 17, но полученная для оптимальной регулярной СС с численным составом N=2. Сравнение рис. 17 и 18 позволяет заключить, что данные уровенные поверхности схожи по своему общему внешнему виду. Можно показать, что такая повторяемость в основных своих чертах сохраняется и для уровенных поверхностей периодичности обзора с помощью СС любого численного состава N>2. Данное обстоятельство автор считает важным дополнительным аргументом в пользу того, что экстремальные РСС близки по своим характеристикам к абсолютно оптимальным, поскольку только при условии их использования мы имеем физически понятный, естественным образом укладывающийся на шкале высот ряд отдельных уровенных линий периодичности.

Остановимся подробнее на вопросе, какие особенности присущи всем этим поверхностям.

Из рис. 17, 18 видно, что уровенная поверхность периодичности всегда разрывна, причем отдельные ее "куски" почти параллельны плоскости "полоса–высота" и находятся на уровнях некоторых периодичностей τ, приблизительно кратных половине суток для односпутникового наблюдения (см. рис. 17), четверти суток для двухспутникового наблюдения (см. рис. 18) и, как можно показать, 1/2N части суток для N–спутникового наблюдения в общем случае.

В направлении уменьшения периодичности τ (увеличения ширины П полос) уровенная поверхность ограничена значениями П, соответствующими нулевому (заданному минимальному) углу места спутника относительно плоскости местного горизонта объектов наблюдения. (Они находятся за пределами рис. 17, 18, справа от них.)

В направлении увеличения периодичности τ обзора (уменьшения П) в каждом сечении

$$m/n=const \quad (39)$$

уровенная поверхность ограничена некоторым предельным значением величины τ, зависящим от периода повторяемости T_{mp} трассы спутника на геосинхронной орбите данной кратности m/n. Однако, в любом хоть сколько-нибудь продолжительном диапазоне высот уровенная поверхность в рассматриваемом направлении становится бесконечной.

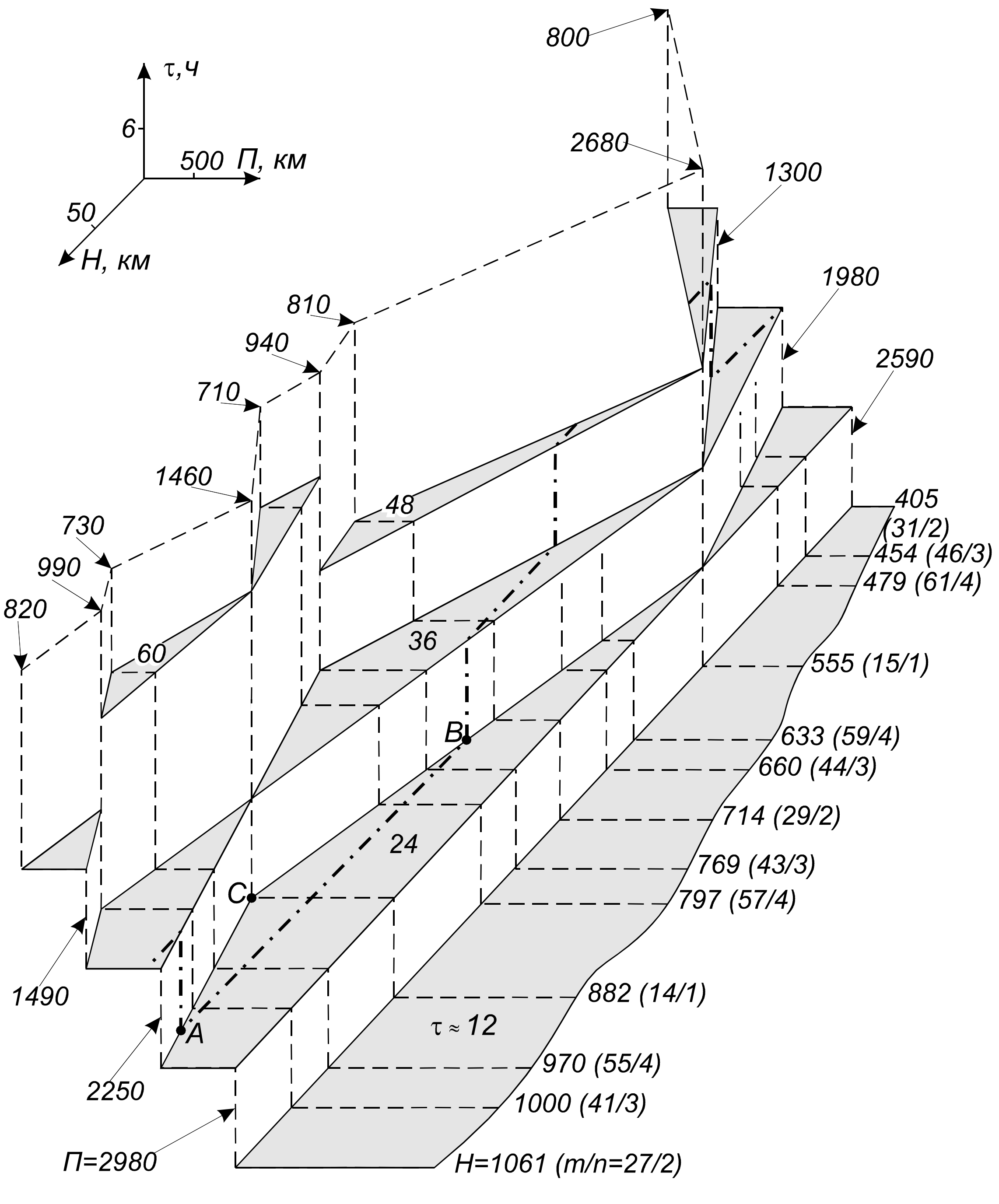

Рис.17. Уровенная поверхность периодичности обзора для одиночного спутника в диапазоне высот *405…1061* км при наклонении $i=85^0$ и широтном поясе наблюдения $0…70^0$.

Данный факт является следствием обсуждавшейся выше возможности сколь угодно большого повышения дискретности множества геосинхронных орбит, имеющихся в фиксированном диапазоне высот, путем увеличения числа n эффективных суток в периоде повторяемости трасс спутников. В пределе с увеличением n «нулевой» ширине П полосы соответствует периодичность обзора $\tau \to \infty$. На рис. 4 уровенная поверхность представлена до величины $\tau \approx 60$ ч, на рис. 5 – $\tau \approx 30$ ч. Прерывистыми линиями на обоих рисунках показаны линии равных кратностей m/n, n=1,2,3,4.

Уменьшение периодичности τ в любом сечении (11) с увеличением ширины П полосы происходит скачкообразно, т.е. функция $\tau(\Pi)$ здесь претерпевает разрывы первого рода. В некоторых случаях подобные скачки в периодичности имеют малую величину и, с целью упрощения очень сложной формы уровенной поверхности, не отражены на рис. 17, 18. В действительности на указанных уровнях периодичности находятся не одиночные “куски”, а группы таких "кусков", расположенных через соответствующие “отступы” по величине τ. Заметим, кстати, что в тех достаточно нечастых случаях, когда абсолютно оптимальные структуры превосходят по периодичности регулярные СС, это выражается в дополнительном дроблении указанной группы “кусков” уровенной поверхности.

Каждый “кусок” реальной уровенной поверхности не строго параллелен плоскости “полоса–высота”, а несколько наклонен к ней под небольшими углами таким образом, что периодичность обзора возрастает с увеличением высоты Н при П=const и остается постоянной с изменением ширины П при Н=const. Такой наклон “кусков” уровенной поверхности объясняется тем, что, как можно показать, различным точкам любого такого “куска” соответствуют однотипные по структуре оптимальные последовательности сеансов наблюдения (потоки наблюдений), которые и определяют наибольший, в том или ином смысле, возможный перерыв (т.е. периодичность τ обзора заданного района). Числовое же выражение этого максимального перерыва зависит от драконического периода T_{dr} обращения спутника таким образом, что для двух различных высот H_1, H_2 с драконическими периодами T_{dr1}, T_{dr2} значения τ_1, τ_2 периодичности соотносятся в соответствии с выражением $\tau_1 : \tau_2 = T_{dr1} : T_{dr2}$, что и предопределяет указанное изменение периодичности для точек каждого “куска”.

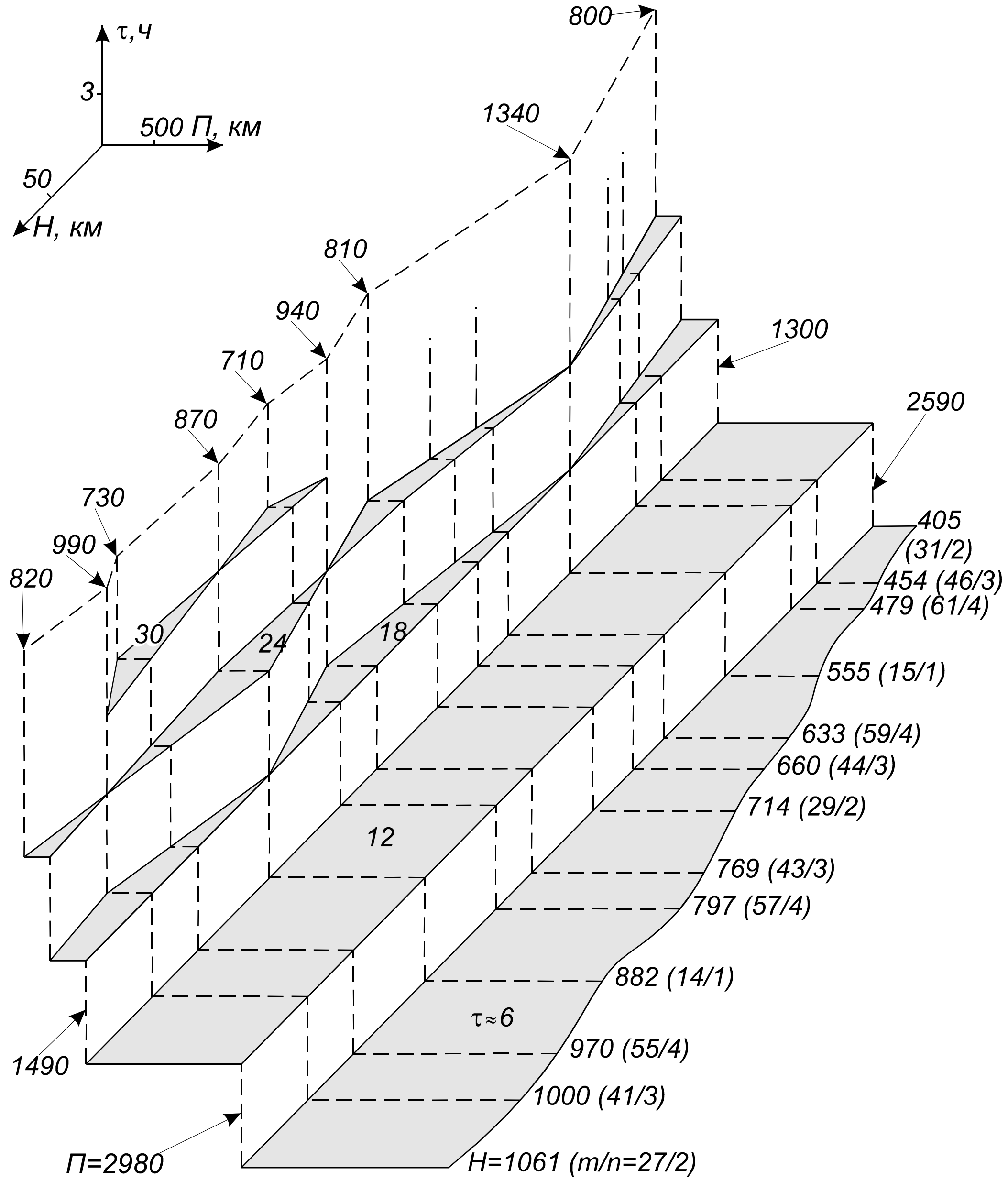

Рис.18. Уровенная поверхность периодичности обзора для оптимальной двухспутниковой системы в диапазоне высот *405…1061* км при наклонении $i=85^0$ и широтном поясе наблюдения $0…70^0$.

Действительно, в свете сказанного понятно, что каждой точке “куска” уровенной поверхности при N=1 (см. рис. 17) соответствует строго определенный поток наблюдений критической точки наблюдаемого района, поскольку такой точке соответствует и единственно определенная односпутниковая орбитальная «структура». Вместе с тем, ситуация при N>1 и, в частности, при N=2 (см. рис. 18) выглядит несколько иначе: каждой точке “куска” уровенной поверхности соответствуют однотипные оптимальные потоки наблюдения критической точки, но соответствовать ей, в общем случае, могут разные спутниковые системы. Регулярная СС, используемая в расчетах уровенных поверхностей, по сути является одной из возможных таких систем. С точки зрения практики это нисколько не ухудшает ситуацию, поскольку нас интересует любая из СС с лучшими характеристиками.

Чрезвычайно интересной, на взгляд автора, особенностью уровенной поверхности периодичности обзора заданного района спутниковой системой любого численного состава является повторяемость ее формы в направлении изменения высоты орбит спутников с периодом, соответствующим изменению кратности k=m/n (как действительного числа) на две единицы: например, изменению кратности суточной (n=1) орбиты с 14/1=14 на 16/1=16 или кратности двухсуточной (n=2) орбиты с 27/2=13,5 на 31/2=15,5. Действительно, последний указанный диапазон изменения кратности $m/n \in \left[27/2, 31/2\right]$ как раз и представлен на рис. 17, 18. На этих рисунках видно, что при соответствующей компенсации увеличения линейного размера ширины П полосы за счет изменения высоты H можно обеспечить совпадение сечений каждой уровенной поверхности на границах указанного интервала.

К другим полезным результатам мы можем прийти, если рассмотрим уровенную поверхность в пространстве параметров «периодичность обзора – ширина полос обзора спутников – наклонение орбит» при фиксированной высоте (кратности) орбит. Такая уровенная поверхность периодичности обзора показана на рис. 6. Данная поверхность является хорошей иллюстрацией выявленной в рамках маршрутной теории общей закономерности локализации минимумов потребных полос обзора спутников в поле указанных параметров, состоящей в следующем.

Минимумы ширины $\Pi_{l\,max}$ полос нечетного l-кратного покрытия достигаются при наклонении (31) для нечетной разности m-n параметров m, n геосинхронной орбиты. При этом под l-кратным покрытием заданного района в задаче периодического обзора понимается такое покрытие, когда в течение периода повторяемости трасс каждая точка наблюдаемого района не менее чем l раз попадает в зоны обзора спутника (спутников).

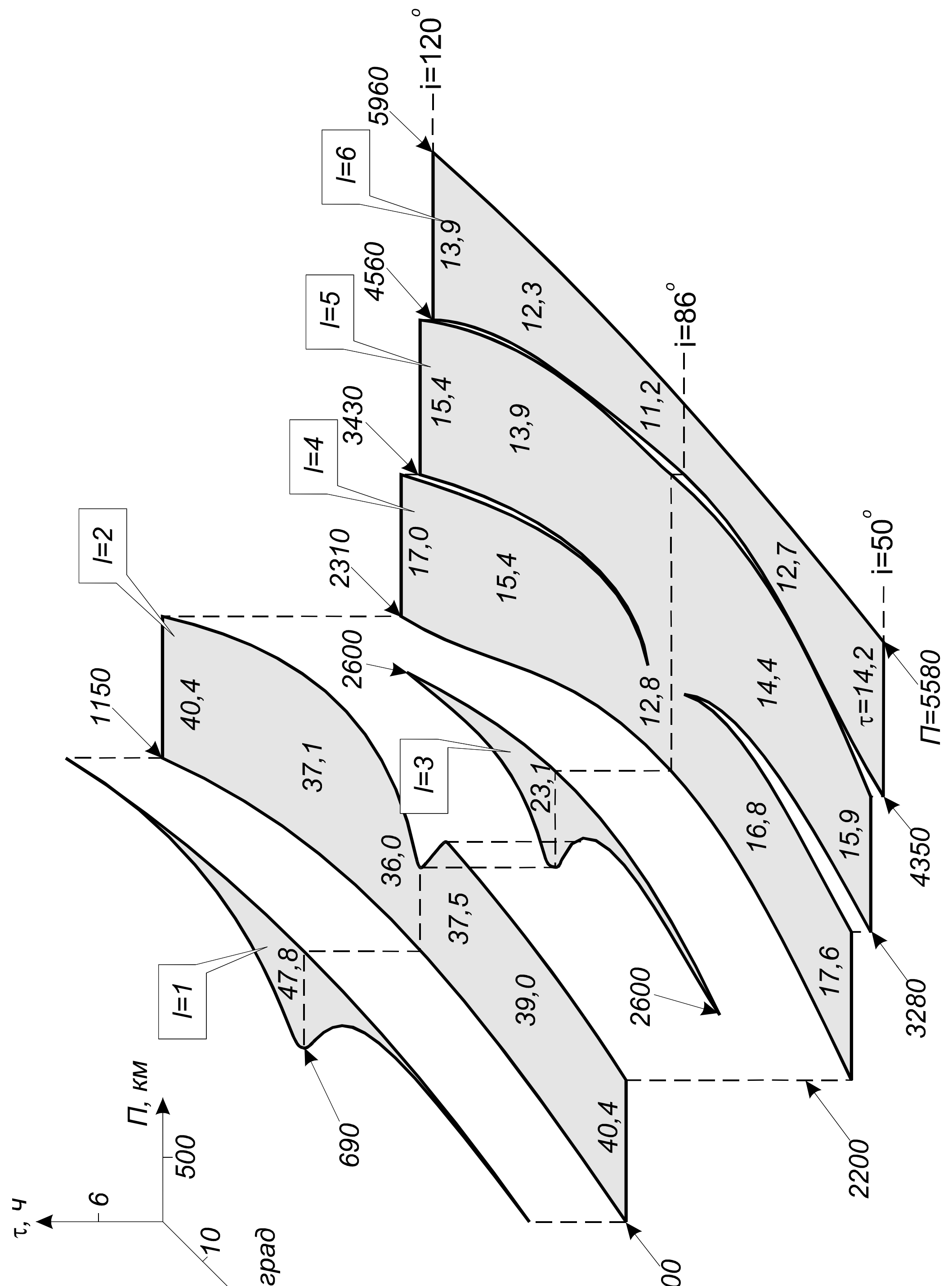

Рис. 19. Уровенная поверхность периодичности обзора широтного пояса $(0,\varphi)$, $\varphi_i = \min(i, \pi - i)$ в пространстве параметров, включающем наклонение орбиты спутника (*m=29, n=2, N=1*).

Указанные минимумы ширины полосы нечетного l-кратного покрытия хорошо видны на приведенной на рис. 19 уровенной поверхности для одного из нечетных чисел m-n=29-2 при наблюдении одиночным спутником широтного пояса $[0,\varphi_i]$, $\varphi_i = \min\{i, \pi - i\}$. В данном случае минимумы имеются на “кусках” уровенной поверхности, соответствующих кратностям l=1, 3, 5 и уровням периодичности $\tau \approx 47{,}8;\ 23{,}1;\ 12{,}4$ ч.

Одновременно из того же рис. 19 видно, что “куски” подобной уровенной поверхности, приходящейся на четные кратности l покрытия, характеризуются изгибом несколько другого профиля – в направлении изменения периодичности τ. Интересно то, что получающийся в этом случае минимум по периодичности приходится на то же самое наклонение $i = i^*$. Так, на рис. 19 при четных кратностях l=2, 4, 6 достигаются минимумы $\tau \approx 36{,}0;\ 12{,}8;\ 11{,}2$ ч. Можно показать, что в случае, когда кратность m/n геосинхронной орбиты выбрана из другого условия: m-n=2k, k=1,2,..., указанный изгиб имеют все “куски” соответствующей уровенной поверхности.

Рассмотренные выше закономерности имеют важное самостоятельное значение для практики баллистического проектирования спутниковых систем. На их основе могут быть созданы комплексы автоматизированного проектирования орбитального построения ССПО по заданным ограничениям, а также разработаны специальные способы построения ССПО, оптимизирующие те или иные их параметры.

12. Conslusions

Elaboration of the route theory has shown that problems of continuous and periodic coverage have strong qualitative difference (that is dictated by the necessity to take into account the Earth rotation at the periodic coverage). Therefore, it is not suitable to look for optimum alternatives of orbital disposition for these two problems within the framework of equal satellite constellation classes. It is proved in [7, 8] that symmetrical systems satisfy necessary optimality condition for the continuous coverage problem. This means that small variations in the position of one satellite in the constellation cannot decrease the main feature of the constellation as a whole. At the continuous coverage, as it is known, such feature is an angular value of the satellite coverage area on the Earth surface. Likewise, taking into account that the said feature for the periodic coverage system is the revisit time, it is demonstrated that the route systems satisfy the necessary optimality condition for the periodic coverage problem.

Power of the route theory is not at all exhausted by revealing the above regular constellations class being although a particular class of satellite systems. The main worth of the theory is that it gives a general method for optimal design of constellation under given requirements using, for instance, optimum regular constellations as a good initial approach. This method allows finding a global optimum for the above stated classical problem of periodic coverage. Practical application of the said method would be impossible without some other theoretical results achieved by the route theory – analytical solution for calculating problem of distribution of revisit time values on the Earth surface for one satellite and multi-satellite route pattern, formulating and proving of several regularities for revisit time as a function of satellite positions in route constellations, etc.

References

1. Vargo, L.G., “Orbital Patterns for Satellite Systems”, Advances in the Astronautical Sciences, Vol. 6, 1960, pp. 709-725.
2. Gobetz, E.W., “Satellite Networks for Global Coverage”, Advances in Astronautical Sciences, Vol. 9, 1961, pp. 134-156.
3. Luders, R.D., “Satellite Networks for Continuous Zonal Coverage”, ARS Journal, Vol. 31, No. 2, 1961, pp. 179-184.
4. Ullock, M.H., Shoen A.H., “Optimum Polar Satellite Networks for Continuous Earth Coverage”, AIAA Journal, Vol. 1, 1963, pp. 69-72.
5. Walker, J.G., “Circular Orbit Patterns Providing Continuous Whole-Earth Coverage”, Royal Aircraft Establishment Technical Report 70211, Nov. 1970.
6. Walker, J.G., “Some Circular Orbit Patterns Providing Continuous Whole Earth Coverage”, Journal of the British Interplanetary Society, Vol. 24, 1971, pp. 369-384.
7. Mozhaev, G.V., “The Problem of Continuous Earth Coverage and Kinematically Regular Satellite Networks, I”, Cosmic Research, Vol. 10, No. 6, Nov.-Dec. 1972, pp. 833-840.
8. Mozhaev, G.V., “The Problem of Continuous Earth Coverage and Kinematically Regular Satellite Networks, II”, Cosmic Research, Vol. 11, No. 1, Jan.-Feb. 1973, pp. 59-69.
9. Walker, J.G., “Continuous Whole Earth Coverage by Circular Orbit Satellite Patterns”, Royal Aircraft Establishment Technical Report 77044, March 1977.
10. Walker, J.G., “Satellite Patterns for Continuous Multiple Whole-Earth Coverage”, Maritime and Aeronautical Satellite Communication and Navigation, IEEE Conference Publication 160, March 1978, pp. 119-122.
11. Barinov, K.N., Burdaev, M.N., Mamon, P.A., “Dynamics and Principles of Designing of Orbital Structures of the Space Vehicle”, Moscow, Mashinostroyenie, 1975, 232 p. (in Russian language).

12. Ballard, A.H., “Rosette Constellations of Earth Satellites”, IEEE Transactions on Aerospace and Electronics Systems, Vol. AES-16, No. 5, Sept. 1980, pp. 656-673.
13. Walker, J.G., “Satellite Constellations”, Journal of the British Interplanetary Society, Vol. 37, 1984, pp. 559-571.
14. Lang, T.J., “Symmetric Circular Orbit Satellite Constellations for Continuous Global Coverage”, AAS Paper, No. 87-499, AAS/AIAA Astrodynamics Conference, Kalispell, Montana, Aug. 1987.
15. Mozhaev, G.V., “Synthesis of Satellite System Orbital Structures: Theoretically-Group Approach”, Moscow, Mashinostroyenie, 1989, 304 p. (in Russian language).
16. Beste, D.C., “Design of Satellite Constellations for Optimal Continuous Coverage”, IEEE Transactions on Aerospace and Electronics Systems, May 1978.
17. Rider, L., “Optimized Polar Orbit Constellations for Redundant Earth Coverage,” The Journal of the Astronautical Sciences, Vol. 33, April-June 1985, pp. 147-161.
18. Draim, J.E., “A Common Period Four-Satellite Continuous Global Coverage Constellation”, AIAA preprint 86-2066-CP, AIAA/AAS Astrodynamics Conference, Williamsburg, VA, Aug. 1986.
19. Lang T.J., Hanson J.M., “Orbital Constellations Which Minimize Revisit Time”, Astrodynamics, Vol. 54, Part 2, 1983, pp. 1071–1086.
20. Bobronnikov, V.T., Shubin, V.I., “Solution Method of the Problem of Satellite Network Design for Earth Surface Periodic Coverage”, Proceedings of XV Tsiolkovsky Readings, Section “Space Flight Mechanics”, Moscow, Nauka, 1981, pp. 52-58 (in Russian language).
21. Mozhaev, G.V., Gutenev, A.A., “Properties of Kinematically Symmetrical Systems in the Earth Periodical Coverage Problem”, Gagarin Scientific Readings on Cosmonautics and Aviation, 1983-1984, Moscow, Nauka, 1985, p. 50 (in Russian language).
22. Schmitt, D., “Low Altitude Satellite Constellations That Minimize Worldwide Revisit Time”, AAS Paper, № 89–379, AAS/AIAA Astrodynamics Conference, Stowe, Vermont, Aug. 1989.
23. Hanson, J.M., Evans, M.J., Turner, R.E., “Designing Good Partial Coverage Satellite Constellations”, Journal of the Astronautical Sciences, Vol. 40, № 2, 1992, pp. 215-239.
24. Lang, T.J., “A Parametric Examination of Satellite Constellations to Minimize Revisit Time for Low Earth Orbits Using a Genetic Algorithm”, AAS Paper, № 01-345, AAS/AIAA Astrodynamics Specialist Conference, Quebec City, Quebec, Canada, Jul.-Aug. 2001.

25. Lang, T.J., “Walker Constellations to Minimize Revisit Time in Low Earth Orbit”, AAS Paper, № 03-178, 13-th AAS/AIAA Space Flight Mechanics Meeting, Ponce, Puerto Rico, Feb. 2003.
26. Lang, T.J., “Streets of Coverage Constellations to Minimize Revisit Time in Low Earth Orbit”, AAS Paper, № 05-154, 15th AAS/AIAA Space Flight Mechanics Meeting, Copper Mountain, Colorado, Jan. 2005.
27. Belokonov, I.V., Borisov, A.V., “Synthesis of Non-symmetrical Systems for Earth Coverage”, Proceedings of XXIII Tsiolkovsky Readings, Section “Problems of Rocket and Space Engineering”, Moscow, USSR Academy of Science, 1990, pp. 85-90 (in Russian language).
28. Belokonov, I.V., Borisov, A.V., “Synthesis of Orbital Structures of One-route Satellite Systems”, Proceedings of XXIV Tsiolkovsky Readings, Section “Problems of Rocket and Space Engineering”, Moscow, USSR Academy of Science, 1990, pp. 31-38 (in Russian language).
29. Saulsky, V.K., “Procedure of the Calculation of the Coverage Periodicity Provided by the Earth Natural Resources Research System”, Earth Research from Space, Journal of USSR Academy of Sciences, No. 5, 1986, pp. 103-112 (in Russian language).
30. Saulsky, V.K., “Method for Optimization of Orbits and Structure of the Satellite Constellations for Earth Twenty-Four-Hour Periodical Coverage”, Earth Research from Space, Journal of USSR Academy of Sciences, No. 1, 1987, pp. 112-121 (in Russian language).
31. Saulsky, V.K., “Optimal Orbits and Structure of Earth Artificial Satellites Systems for Earth Periodical Coverage”, Earth Research from Space, Journal of USSR Academy of Sciences, No. 2, 1989, pp. 104-115 (in Russian language).
32. Byrkov, B.P., Razoumny, Yu.N., “Solution of Problem of the Earth Multiple Coverage by Earth Artificial Satellites Swath”, Earth Research from Space, Journal of Russian Academy of Sciences, No. 1, 1992, pp. 62–71 (in Russian language).
33. Byrkov, B.P., Razoumny, Yu.N., “Analytical Method to Define the Earth Coverage Periodicity by Satellite of Earth Artificial Satellite System // Earth Research from Space, Journal of Russian Academy of Sciences, No. 2, 1992, pp. 91–97 (in Russian language).
34. Byrkov, B.P., Golovatenko-Abramov V.I., “Optimal Ballistic Construction of Earth Artificial Satellite System of the Earth Surface Areas Periodical Coverage”, Earth Research from Space, Journal of Russian Academy of Sciences, No. 6, 1993, pp. 76–80 (in Russian language).

35. Razoumny, Yu.N., “Regular Multiple-Route Satellite Constellations for Earth Periodical Coverage”, Proceedings of XV Scientific Readings on Cosmonautics, Section “Problems of Space Vehicle Orientation and Motion Control”, Moscow, USSR Academy of Science, 1991, p. 29 (in Russian language).
36. Razoumny, Yu.N., “Optimal Structure of One-Route Satellite System for Earth Periodical Coverage”, Proceedings of XXVI Tsiolkovsky Readings, Section “Problems of Rocket and Space Engineering”, Moscow, Russian Academy of Science, 1992, pp. 79-84 (in Russian language).
37. Razoumny, Yu.N., “Method of Parameters Optimization of the One-Route Earth Artificial Satellite Systems for the Earth Periodical Coverage”, Earth Research from Space, Journal of USSR Academy of Sciences, No. 5, 1991, pp. 39-46 (in Russian language).
38. Razoumny, Yu.N., “Optimization of Designed Ballistic Parameters of Satellite Systems for the Earth Periodical Coverage”, Earth Research from Space, Journal of Russian Academy of Sciences, No. 1, 1993, pp. 48-58 (in Russian language).
39. Razoumny, Yu.N., Ermachenkov A.V., “Analytic Method of Determining the Periodicity of Imaging the Earth by Satellite System and the Rational Orbit Designe for the Earth Resources Observation”, Earth Research from Space, Journal of Russian Academy of Sciences, No. 2, 1995, pp. 57-70 (in Russian language).
40. Razoumny, Yu.N., “Concept of Ballistic Design of Multi–Satellite Networks for Earth Periodic Coverage Built Around the Route Orbital Structures”, Proceedings of the Second International Aerospace Congress “IAC`97”, Moscow, Russia, Aug.-Sep. 1997, Vol. 2, 1999, pp. 293-296.
41. Razoumny, Yu.N., “Synthesis of Satellite Constellations for Periodical Coverage”, Moscow, Publishing House of the Bauman Moscow State Technical University, 2000, 104 p. (in Russian language).
42. Razoumny, Y.N., “Route Theory for Optimal Design of Satellite Constellations to Minimize Revisit Time in Low Earth Orbits”, Proceedings of the 56-th International Astronautical Congress, October 17-21, 2005, Fukuoka, Japan, IAC-05-C1.P.02, 11 p.
43. Razoumny, Y.N., Razoumny, V.Y. “Constellation Design for Earth Periodic Coverage in Low Orbits with Minimal Satellite Swath”, Proceedings of the 58-th International Astronautical Congress, September 24-28, 2007, Hyderabad, India, IAC-07-C1.7.09, 19 p.
44. Robertson, R.L., “Low-Cost Earth Observation from Space”, AIAA Paper, No. 79–0126, 17-th Aerospace Sciences Meeting, New Orleans, Louisiana, Jan. 1979, 8 p.

45. Razoumny, Y.N., Alekseev, E.V., et al. Method of observation of Earth surface from space. Invention with a patent of Russian Federation No. 2076059 dd. 1997 (with priority from Sep. 9, 1988).
46. Razoumny, Y.N., Method of observation of Earth surface from space. Invention with a patent of Russian Federation No. 2058917 dd. 1996 (with priority from Apr. 1, 1991).
47. Razoumny, Y.N., Alekseev, E.V., et al. Method of observation of Earth surface from space. Invention with a patent of Russian Federation No. 2075862 dd. 1994 (with priority from Apr. 11, 1994).
48. Razoumny, Y.N., Gusev, Y.G., et al. Method of observation of Earth surface from space. Invention with a patent of Russian Federation No. 2118273 dd. 1995 (with priority from June 6, 1995).

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